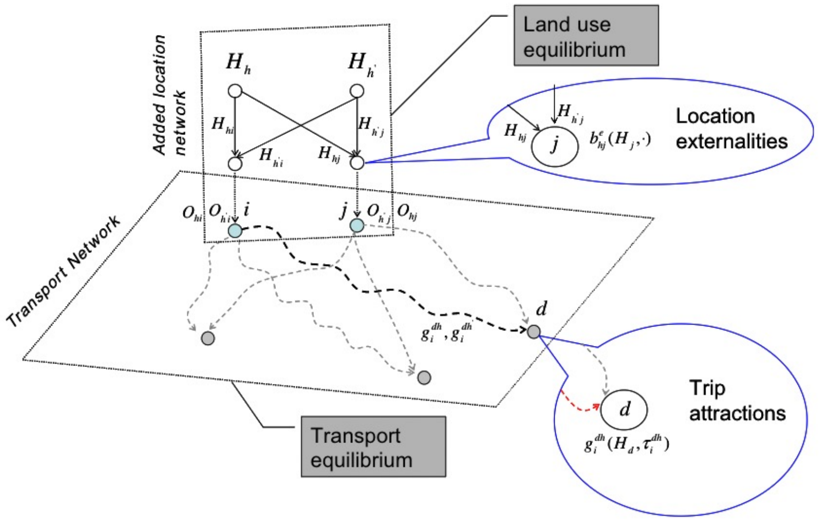
Reading seminar for international students #3

An integrated behavioral model of the land-use and transport systems with network congestion and location externalities

Bravo, M., Briceño, L., Cominetti, R., Cortés, C. E., & Martínez, F. (2010).
Transportation Research Part B: Methodological, *44*(4), 584-596.



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Summary

Modeling big urban areas for planning purpose

- **Dynamic interaction between land-use and transport** are jointly analyzed as long-term equilibrium
- Two types of externality (**road congestion** and **location externalities**) are newly considered

Discrete choice-based

- The location, travel decisions, and route choices are represented by <u>logit models</u>.
- consumers optimize their combined residence and transport options represented as paths in an <u>extended network</u>

Existence of equilibria

- model as a **fixed-point problem**, establishing the existence of equilibria

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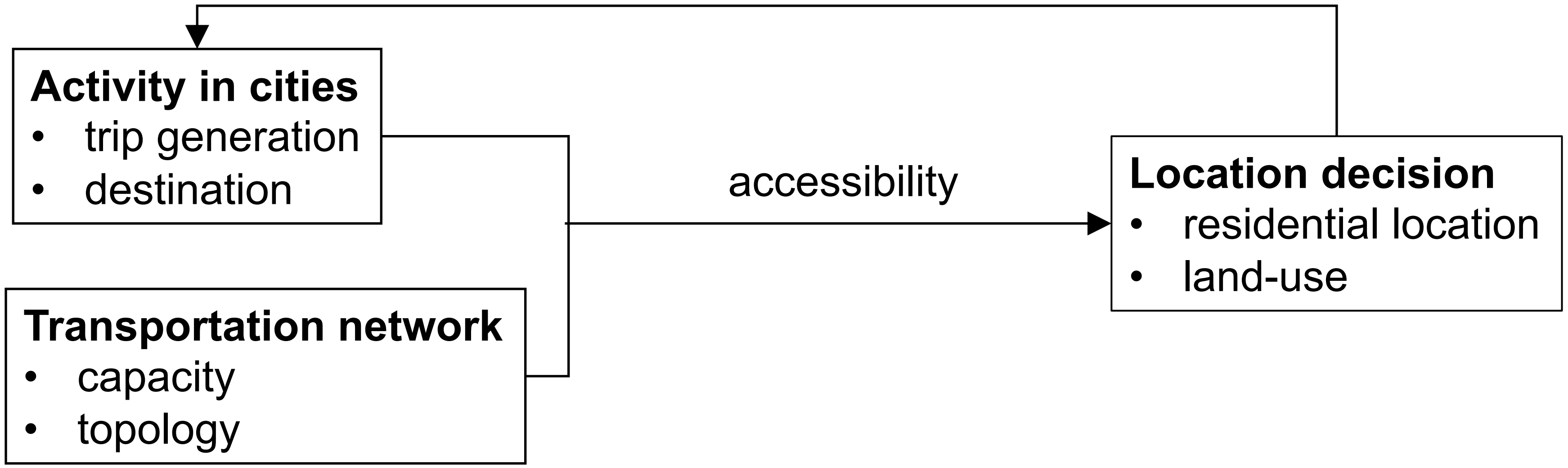
Introduction

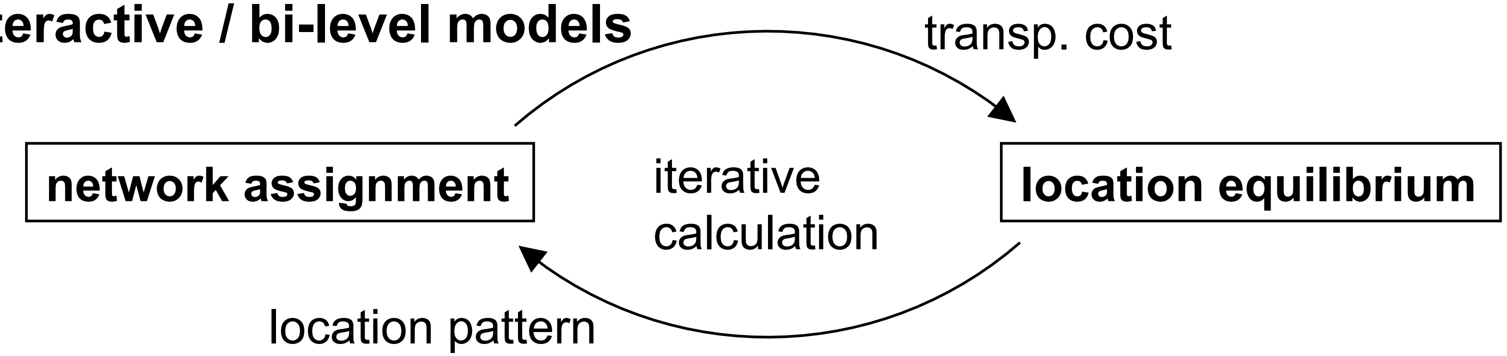
Motivation: To properly represent the **interaction** between **the transportation system** and **the spatial distribution of residential and non-residential activities**.

= dynamic interaction of land-use and transport systems



Novelty of this study

- **Interactive / bi-level models**



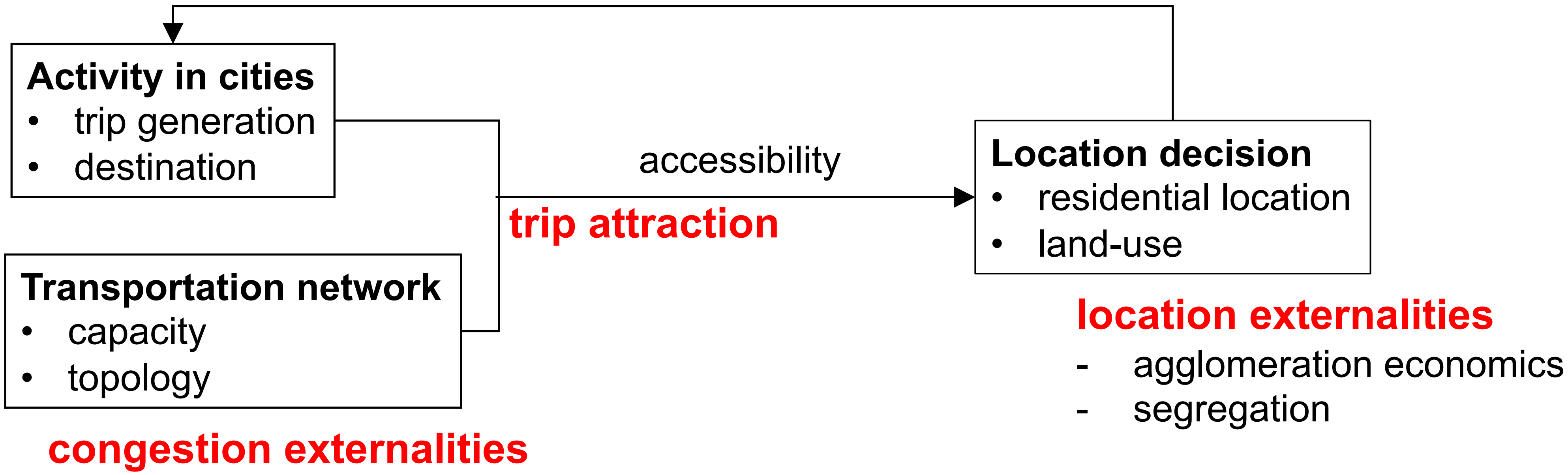
× do not analyze the existence and uniqueness of equilibria

× convergence of the iterations

× high computational cost

- **This study**
  simultaneously solve the internal conflicts within the transport and land-use sub-systems along with their interactions

Modeling the externalities



Goal of this study:
To search for sufficient conditions to ensure the existence of equilibrium in land-use and transport system considering those interactions.

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Literature review

Interaction between land-use and transportation

× interactive / bi-level models

× route-based traffic assignment does not work with large NWs.
→ arc-based stochastic traffic assignment
= Markovian traffic equilibrium (Baillon and Cominetti, 2008)

- At each intermediate node, the traveler decide the next arc based on a discrete choice model in order to minimize the expected travel time to destination

× no analysis of equilibrium on land prices nor on externalities
→ adopts the **Random Bidding and Supply Model** (Martínez and Henríquez, 2007)

- auction mechanism under the best bid rule
- the willingness-to-pay for each location as proposed by Alonso (1965)

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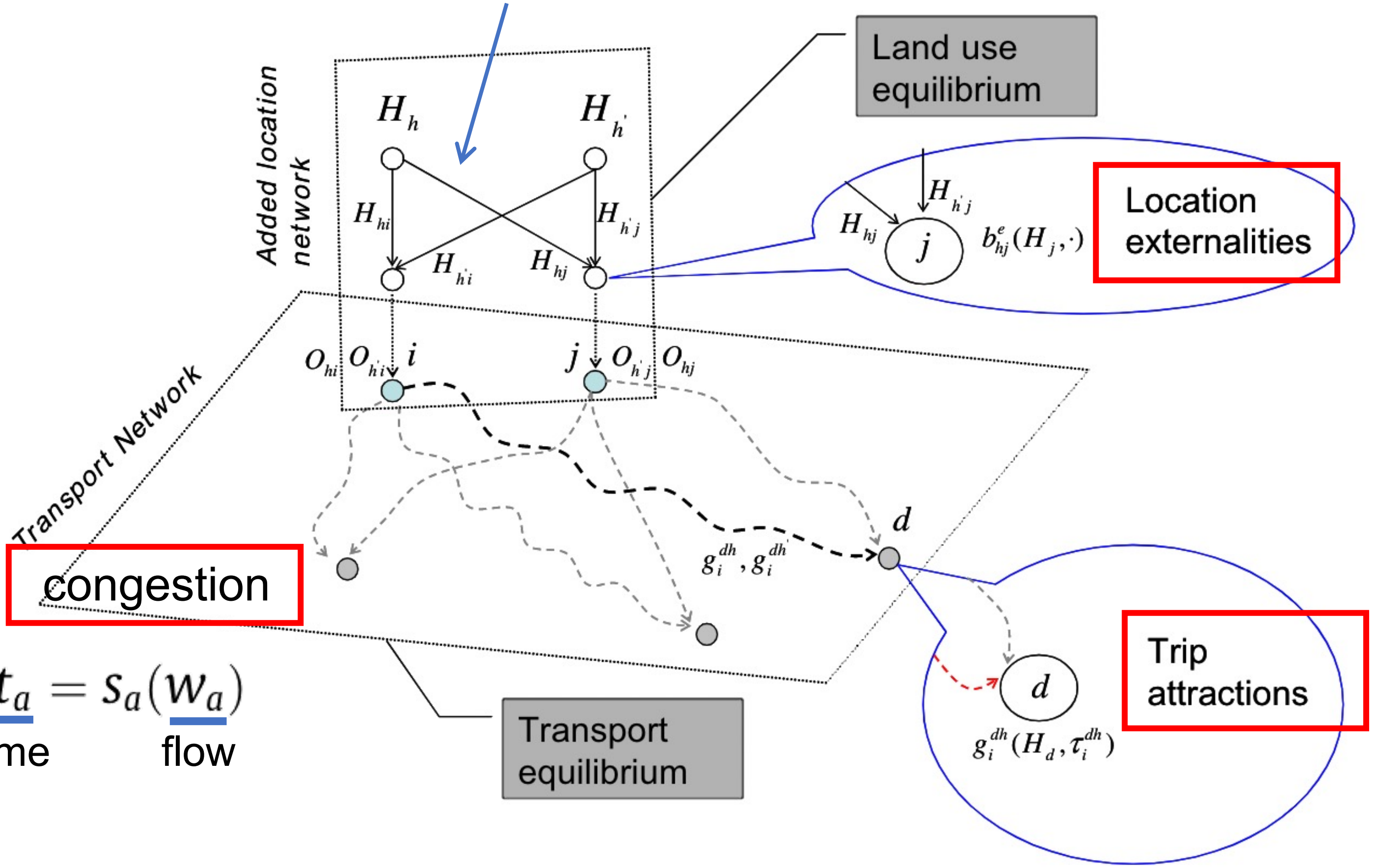
General framework

- Urban system as an extended network

Arrows in location NW just represent interaction, not movement / relocation



$t_a = s_a(w_a)$

travel time — flow

WTP for agent type h for location i — monetary disutility

$$b_{hi} = b^e_{hi}(H, O, \alpha, t) - b^u_h$$

Externality term depending on

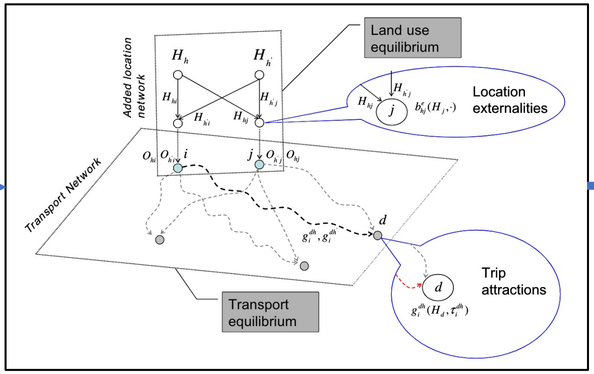
- H: # agents in zone i
- O: # trips form zone i
- α: lagrangean multiplier
- t: travel time

Fixed point algorithm

system: LU&T model

H: # agents in zone i

H → input



output → $\widetilde{H}$

If the input H is still $\widetilde{H}$ after going through the system, equilibrium attained and H is fixed point.

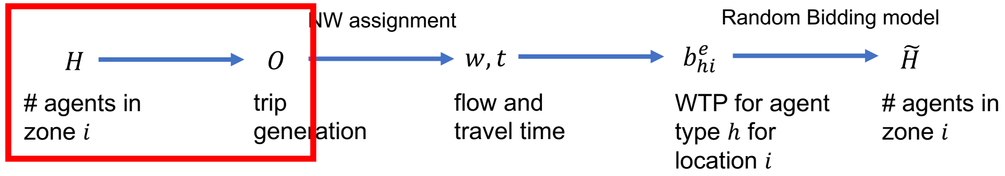
$$H = f(\widetilde{H}, \dots)$$

more details

H → O —NW assignment→ w, t → b_{hi}^{e} —Random Bidding model→ $\widetilde{H}$

- H: # agents in zone i
- O: trip generation
- w, t: flow and travel time
- b_{hi}^{e}: WTP of agent type h for location i
- $\widetilde{H}$: # agents in zone i

Formulation (1)



- Trip generation

trip rate per agent

$$O_{hi} = N_{hi} H_{hi} + \delta_{hi}$$

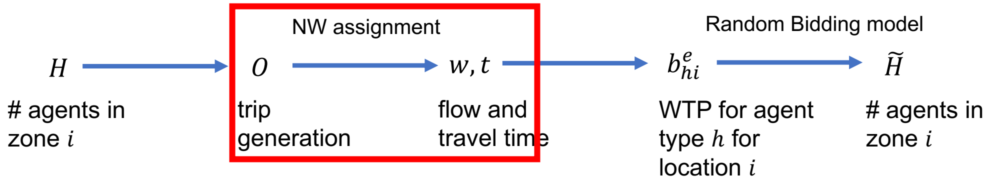
trips by agent type *h* from zone *i*

agent type *h* in zone *i*

minimum constant trip generation (>0)

Very simple.

Formulation (2)



- Trip distribution = maximum entropy model (usual model for trip distribution)
  - Generated trips are distributed so as to maximize entropy of the system

cost of choosing d

$$(P)\quad \min_{g} \sum_{i\in I}\sum_{d\in D}\sum_{h\in C} c_i^{dh} g_i^{dh} + \sum_{h\in C}\frac{1}{\mu_h}\sum_{j\in I}\sum_{d\in D} g_j^{dh}(\ln g_j^{dh} - 1)$$

trips from zone i to d

Entropy term

s.t.

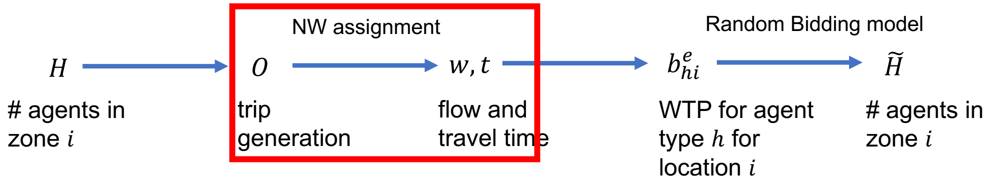
$$\sum_{d\in D} g_i^{dh} = O_{hi}$$

trips by agent type *h* from zone *i*

Lagrangean dual

$$(D)\quad \min_{\alpha} \sum_{h\in C}\sum_{i\in I} O_{hi}\alpha_{hi} + \sum_{h\in C}\frac{1}{\mu_h}\sum_{i\in I}\sum_{d\in D} \exp[-\mu_h(c_i^{dh} + \alpha_{hi})].$$

Formulation (3)



- Trip assignment = Markovian traffic equilibrium (MTE)
  - passengers travel to their destination by a recursive procedure
  - an exit arc is randomly selected at every intermediate node, using a discrete choice model that seeks to minimize the expected time-to-destination.
  - MTE is characterized as the optimal solution of

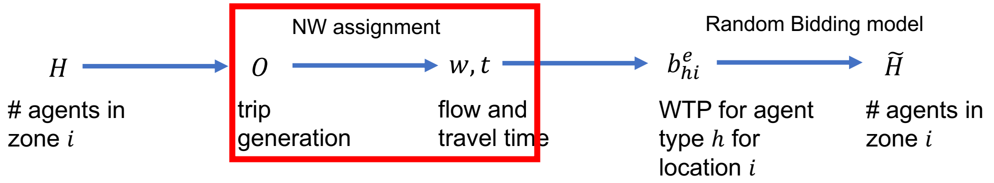
$$\min_{t} \Phi(t) = \sum_{a \in A} \int_{t_a^0}^{t_a} s_a^{-1}(z)dz - \sum_{h \in C} \sum_{\substack{d \in D \\ i \neq d}} g_i^{dh} \tau_i^{dh}(t)$$

s_a^{-1}: travel time function

g_i^{dh}: **trips** from zone i *to* d

$\tau_i^{dh}(t)$: **expected travel time to d**
= (travel time of link a) + (expected travel time to d)

Formulation (4)



- Combining trip distribution and assignment model, joint distribution / network-assignment equilibrium is represented as follows,

$$\min_{\alpha,t} \Phi(\alpha,t) = \sum_{a\in A} \int_{t_a^0}^{t_a} s_a^{-1}(z)dz + \sum_{h\in C}\sum_{i\in I} O_{hi}\alpha_{hi} + \sum_{h\in C}\frac{1}{\mu_h}\sum_{i\in I}\sum_{d\in D}\exp\left[-\mu_h(c_i^{dh}(t,H)+\alpha_{hi})\right]$$

$$c_i^{dh}(t,H) = \tau_i^{dh}(t) - \gamma_d(H)$$

expected travel time to d — the benefits from land-use attributes at location d

(Optimality condition)

$$s_a^{-1}(t_a) = \sum_{d\in D; h\in C} v_a^{dh} = w_a \qquad v_a^{dh} = \sum_{i\in I} g_i^{dh}\frac{\partial \tau_i^{dh}}{\partial t_a}$$

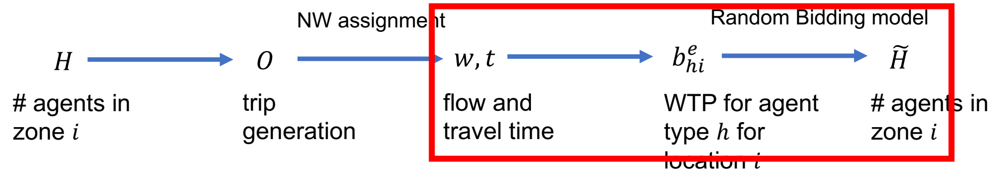
$$g_i^{dh} = O_{hi}\cdot P_{d/ih} \quad \text{,where} \quad P_{d/ih} = \frac{\exp[-\mu_h(\tau_i^{dh}(t)-\gamma_d(H))]}{\sum_{k\in D}\exp[-\mu_h(\tau_i^{kh}(t)-\gamma_k(H))]}$$

Optimality condition of MTE

$$\alpha_{hi} = \frac{1}{\mu_h}\ln\left(\frac{1}{O_{hi}}\sum_{d\in D}\exp[-\mu_h(\tau_i^{dh}(t)-\gamma_d(H))]\right)$$

Optimality condition of MTE

Formulation (5)



- Best-bid auction and location mechanism
  → real estate transactions by an auction mechanism under the best bid rule.
  - Prob. for an agent h to set the highest bid and get located at zone i

(logit form) $$P_{h/i} = \frac{\exp[\theta_i b_{hi}]}{\sum_{g \in C} \exp[\theta_i b_{gi}]}.$$

WTP of agent type h for zone i

WTP for agent type h for location i

monetary disutility

$$b_{hi} = b^e_{hi}(H, O, \alpha, t) - b^u_h$$

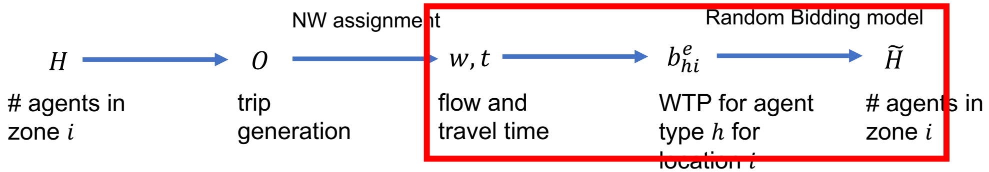
Externality term depending on
- H: # agents in zone i
- O: # trips form zone i
- α: lagrangean multiplier
- t: travel time

  - The total number of agent of type h located at zone i is

$$\tilde{H}_{hi} = S_i \frac{\exp[\theta_i (b^e_{hi} - b^u_h)]}{\sum_{g \in C} \exp[\theta_i (b^e_{gi} - b^u_g)]} \geqslant 0.$$

real estate supply

Formulation (6)



- Best-bid auction and location mechanism
  - Since $\sum_{i\in I}\tilde{H}_{hi} = H_h$ for all $h \in C$,

$$\sum_{i\in I} S_i \frac{\exp[\theta_i(b^e_{hi} - b^u_h)]}{\sum_{g\in C}\exp[\theta_i(b^e_{gi} - b^u_g)]} = H_h \ \forall h \in C$$

  - This is the optimality condition of the following problem

$$\min_{b^u} \Gamma(b^u) = \sum_{h\in C} H_h b^u_h + \frac{1}{\theta_i}\sum_{i\in I} S_i \ln\left(\sum_{h\in C}\exp[\theta_i(b^e_{hi} - b^u_h)]\right).$$

  - If $b^u_1 = 0$, the above problem is convex, and we can recover unique vector b_u

Existence, uniqueness, and convergence

Assumptions

$$(\Pi_0)\begin{cases}\bullet \text{ the functions } \varphi_i^{dh}(\cdot) \text{ are of class } C^3 \text{ and belong to the class } \xi \text{ with } \varphi_d^{dh}(\cdot)\equiv 0,\\ \bullet\ s_a(\cdot) \text{ is strictly increasing and continous with } \lim_{x\to\infty} s_a(x)=\infty,\\ \bullet\ t_a^0 = s_a(0) \geqslant 0 \text{ and } \varphi_i^{dh}(t^0)>0 \text{ for all } i\neq d\end{cases}$$

ex) BPR function

$$(\Pi_1)\ \sum_{i\in I} S_i = \sum_{h\in C} H_h,$$ total real estate supply = demand

$$(\Pi_2)\ b_1^u = 0.$$ normalization

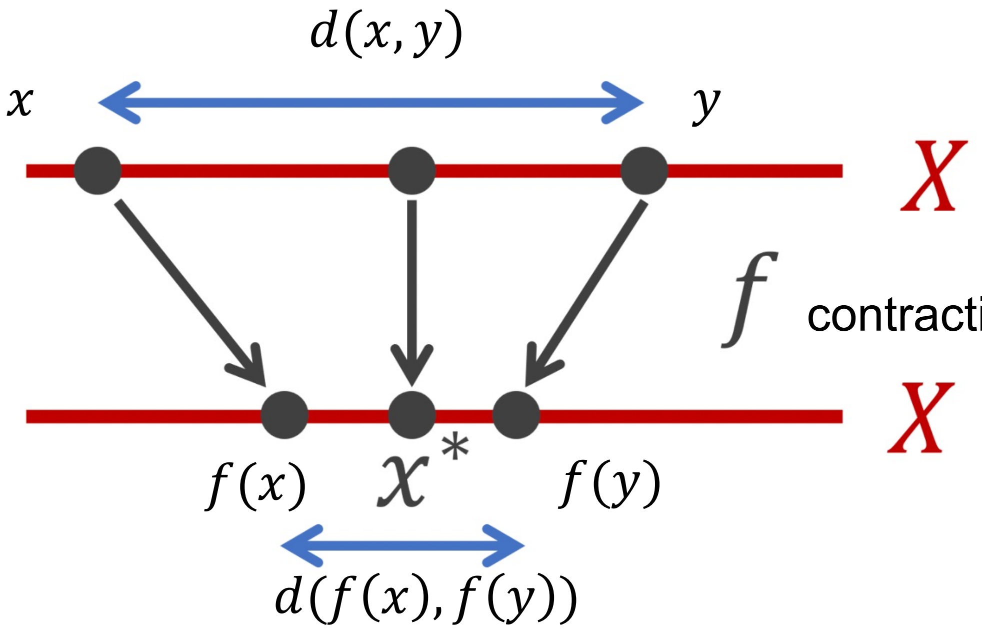
Theorem 1. Assuming (Π_0)-(Π_2), there is at least one LU&T equilibrium ← Brower's Fixed-Point Theorem

Lemma 3. There exists $\theta_c > 0$ such the map $\mathrm{H} \longmapsto \Theta(H)$ is a **contraction** from K to itself as long as $\theta \in (0, \theta_c)$ for all $i \in I$

Existence, uniqueness, and convergence

Theorem 2. Assuming (Π_0)-(Π_2) and $\theta \in (0, \theta_c)$ for all $i \in I$, there is a unique integrated LU&T equilibrium which can be **computed by the convergent fixed-point iteration** $H^{k+1} = \Theta(H^k)$.

← By Banach Fixed-Point Theorem



contraction mapping

$$d(x,y) \leq k \cdot d\big(f(x), f(y)\big), 0 \leq k \leq 1$$

By repeatedly calculating $\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{f}\big(\boldsymbol{f}(\boldsymbol{x})\big)$, …, any start point $\boldsymbol{x}$ converges to $\boldsymbol{x}^* = \boldsymbol{f}(\boldsymbol{x}^*)$

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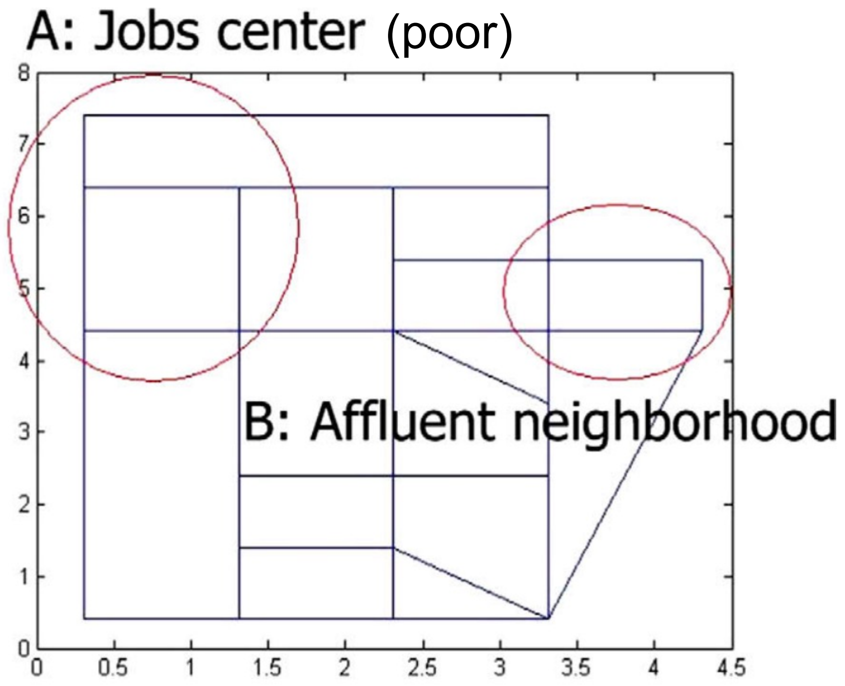
Simulations

- Simulate segregation of rich people and poor people



$$b^{e}_{hi}(H, O, \alpha, t) = z_{hi} + v_h \ln O_{hi} + \rho_h \alpha_{hi} + \Omega_h(H_{\cdot i})$$

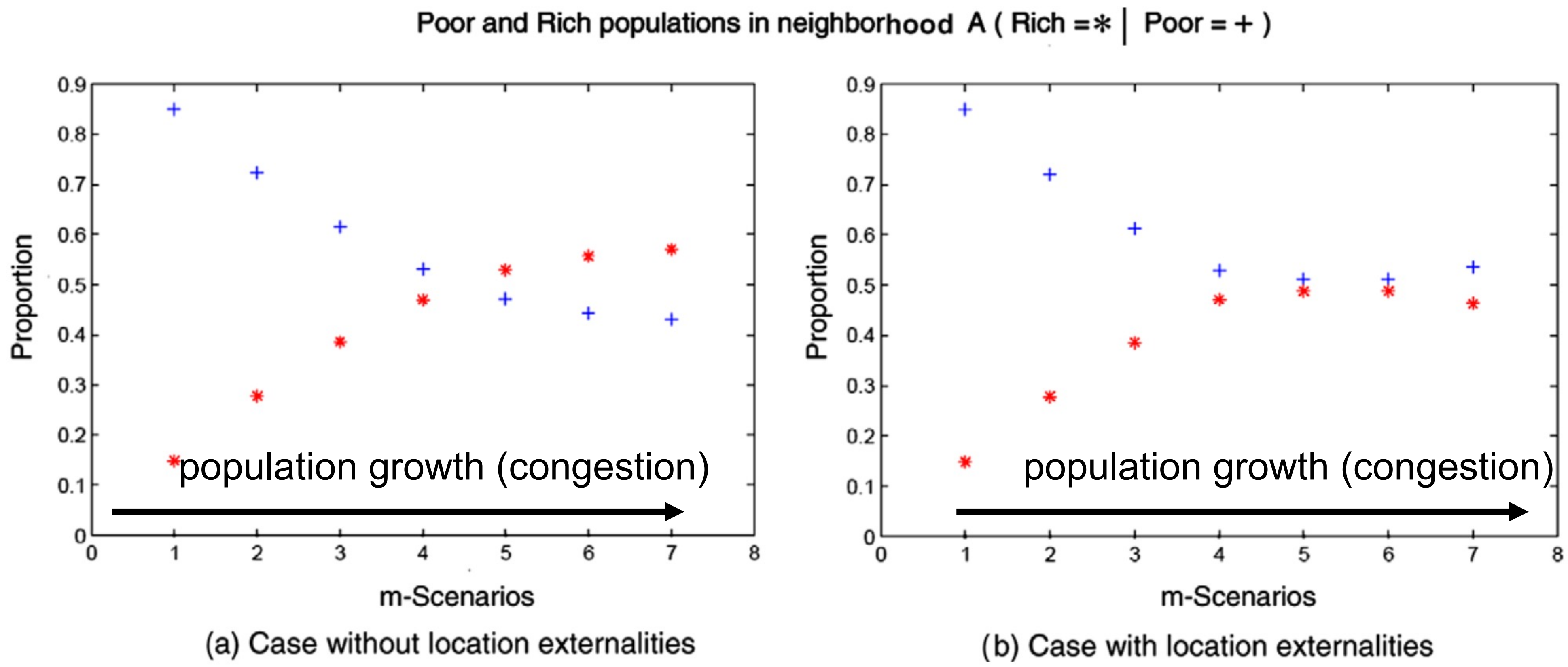
Externality term for WTP

- z_{hi}: zone's features
- H: # agents in zone i
- O: # trips form zone i
- α: lagrangean multiplier
- t: travel time

consumer's valuation of accessibility

like or dislike for other agents

Results



Rich people move to neighborhood A (job center) with increasing congestion

Segregation induces a higher preference of rich families for neighborhood B

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Conclusions and further research

- Integrate the land-use and the transportation systems focusing on **transport and location externalities**
- **A fixed-point model** is used to prove existence of equilibria, and to identify a mild condition on the dispersion of consumers' bids that guarantees uniqueness and convergence of a fixed-point iteration.

Further research

- flexibilities in trip generation model
- inclusion of public transportation
- considering delayed effect of slow-moving variables (infrastructure planning)

Related to my research

Analysis of multi-dimensional planning and urban formation

- Public transportation planning (railway or bus route optimization)
- Urban facility relocation/maintenance problem
- Residential location choice model
- Residents' activity choice

→ Long term equilibrium should be calculated

- Sophisticated trip generation / distribution model with activity models
  - Each sub-model is simple and orthodox
    → required for strict discussion on existence, uniqueness, and convergence of solutions
    → there should be a chance to develop more sophisticated and joint modeling?
- The idea of **fixed point** can be useful when we consider **interaction of multiple players**.
  - We can discuss "equilibrium" theoretically
  - Convergence and uniqueness of equilibrium is important for my research

Reference

This paper is based on the previous work by the authors.

Briceño, L., Cominetti, R., Cortés, C. E., & Martínez, F. (2008). An integrated behavioral model of land use and transport system: a hyper-network equilibrium approach. *Networks and Spatial Economics*, *8*, 201-224. https://link.springer.com/article/10.1007/s11067-007-9052-5

Trip distribution model

https://ocw.tudelft.nl/wp-content/uploads/2.2-Trip-distribution.pdf