Optimal Transport Networks in Spatial Equilibrium

Pablo D. Fajgelbaum, Edouard Schaal (2019)
NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138

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Pankaj Kumar

Table of Contents

Objective

- Find the optimal infrastructure investments allocations across regions
- Fit a quantitative trade model to data on the geographic distribution of economic activity, and find the impact of change in trade costs between specific locations
- Develop a framework for a Neoclassical Trade Model with labour mobility
  - To study optimal transport networks in general equilibrium spatial models
- Apply the framework to European road networks to assess:
  - Aggregate and regional impacts of optimal infrastructure growth
  - Inefficiencies of observed networks, and
  - Optimal placement of roads as a function of observable regional characteristics.

Background

- **Monge's Optimal Transport Problem**
  - Optimal way to transport one distribution of mass to another (optimal assignment)
- **Kantorovich's Optimal Transport Problem**
  - Mathematical optimization problem to find the optimal way to transport mass from one location to another such that the total cost of transportation is minimized
  - An optimal transport plan can be found by maximizing the dual problem, which involves finding a Lipschitz function that approximates the cost function
  - Transport cost: primitive
  - Optimal Transport Plan: outcome
- Trade costs partly determine spatial distributions of prices, real incomes, and trade flows. What if Trade cost is an outcome and is used in counterfactuals to recommend welfare policies?

Related Literature

- **Trade models having counterfactuals w.r.t. Trade costs**
  - Eaton & Kortum (2002): quantitative version of Ricardian trade model
  - Anderson & van Wincoop (2003): quantitative Armington trade model
- **Using counterfactuals w.r.t. shipping costs over least cost routes**
  - Allen & Arkolakis (2014)- aggregate effect of the US highway system
  - Donaldson & Hornbeck (2016)- historical impact of railroads on the US economy
  - Alder (2019)- counterfactual transport networks in India
  - Nagy (2016)- impact of development of US railroads on city formation
- **Optimal Transport Methods in Economics, Alfred Galichon (2016)**
  - Optimal flow problem ~ Optimal route problem
  - Mapping sources with fixed supply to sinks with fixed demand
  - Transport costs as primitive (spatial price distribution, real income, trade flows)
- **Commonality: Optimal transport problem**
  - Trader's problem of choosing least cost routes across pairs of locations

Innovations in Approach

- **Solve global optimization over a space of networks** in neoclassical framework
- **Trade cost as outcome, not primitive**. Use counterfactuals w.r.t. trade costs to analyse polices.
- **Model applicable to any country**
  - Planners can use to analyse the transport infrastructure investments
  - Previous studies just analysed impact of trade costs on one economy
- **Applicable for all cases of congestion**
  - Framework rather designed for strong cases of congestion
  - No congestion is just a special case (e.g. gravity trade models)
- **Locations are arranged on a graph** and goods can only be shipped through connected locations subject to transport costs
  - how much is shipped (e.g., congestion or decreasing returns to shipping technologies)
  - how much is invested in infrastructure (e.g., number of lanes or quality of road).

Model Framework

Assumptions:

- The per-unit cost of shipping is increasing in the quantity of commodities shipped allowing for decreasing returns in the shipping sector, i.e., congestion.
  - The more is shipped, the higher the per-unit shipping cost.
- The per-unit cost of shipping is decreasing with the increase in infrastructure.
- The multiplier of the conservation of flows constraint, P_n reflects society's valuation of a marginal unit of good n in location j, i.e. the price of good in the decentralized allocation.
- Building infrastructure requires a resource in fixed aggregate supply K, which can be freely shipped across locations and cannot be used for other purposes. Hence, the opportunity cost of building infrastructure in any location is only foregoing infrastructure elsewhere.

Model Framework

Definitions of terms:

- Utility of individual worker, $u = U(c,h)$
- Per-capita consumption of traded goods, $c_j = C_j / L_j$
- Aggregate demand of traded goods in location j, $C_j = D_j\left(D_j^1, \dots, D_j^N\right)$
- Fixed supply $V_j = \left(V_j^1, \dots, V_j^M\right)$; $m = 1,\dots,M$ primary factors in location j
- Output of sector n in location j, $Y_j^n = F_j^n\left(L_j^n, V_j^n, X_j^n\right)$ where $V_j^n = \left(V_j^{1n}, \dots, V_j^{Mn}\right)$ and $X_j^n = \left(X_j^{1n}, \dots, X_j^{Nn}\right)$
- Locations J are arranged on an undirected graph (J, ε), where ε denotes the set of edges (i.e., unordered pairs of J). For each location j there is a set $N(j)$ of connected locations, or neighbours. Goods can only be shipped through connected locations; i.e., goods shipped from j can be sent to any $k \in N(j)$, but to reach any $k' \notin N(j)$, they must transit through a sequence of connected locations.

Model Framework

Definition 1: The planner's problem with immobile labour

$$\boldsymbol{W = \max_{c_j, h_j, \{I_{jk}\}k \in N(j), \{D_j^n, L_j^n, V_j^n, X_j^n, \{Q_{jk}^n\}k \in N(j)\}n} \sum_j w_j L_j U(c_j, h_j)}$$

subject to:

(i) availability of traded commodities, $c_j L_j \leq D_j \left(D_j{}^1, \ldots\ldots, D_j{}^N\right)$ for all j;

and availability of non-traded commodities, $h_j L_j \leq H_j$ for all j;

(ii) the **balanced-flows constraint**,

$$D_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in N(j)} (1 + \tau_{jk}(Q_{jk}^n, I_{jk})) Q_{jk}^n \leq F_j^n\left(L_j^n, V_j^n, X_j^n\right) + \sum_{i \in N(j)} Q_{ij}^n$$

for all j, n

Consumption + Intermediate use + Exports ≤ Production + Imports

Model Framework

(iii) the **network-building constraint**, $\sum_{j} \sum_{k\in N(j)} \delta^{I}_{jk} I_{jk} \leq K$,

subject to a pre-existing network, $0 \leq I\_{jk} \leq I_{jk} \leq \bar{I}_{jk} \leq \infty$ for all $j, k \in N(j)$;

(iv) local labour-market clearing, $\sum_{n} L_j^n \leq L_j$ for all j;

and local factor market clearing for the remaining factors, $\sum_{n} V_j^{mn} \leq V_j^m$ for all j and m;

(v) non-negativity constraints on consumption, flows, and factor use,

$c_j, h_j, C_j^m \geq 0$ for all $j \in N(j)$ and n

$Q_{jk}^n \geq 0$ for all $j, k \in N(j)$ and n

$L_j^n, V_j^{mn} \geq 0$ for all j, m and n

Model Framework

- **Definition 2**: The planner's problem with fully mobile labour:

$$\boldsymbol{W} = \max_{u, c_j, h_j, \{I_{jk}\}_{k \in N(j)}, \{D_j^n, L_j^n, V_j^n, X_j^n, \{Q_{jk}^n\}_{k \in N(j)}\}_n} \boldsymbol{u}$$

subject to restrictions (i)-(v) above; as well as:

(vi) free labour mobility, $L_j u \leq L_j U(c_j, h_j)$ for all j;

(vii) aggregate labour-market clearing, $\sum_j L_j \leq L$.

- Case without labour mobility ~ International trade models
- Case with labour mobility ~ Urban economics model with a single homogenous tradable good

Model Framework

- The planner's problem of Definition 1 can be expressed as nesting three problem:

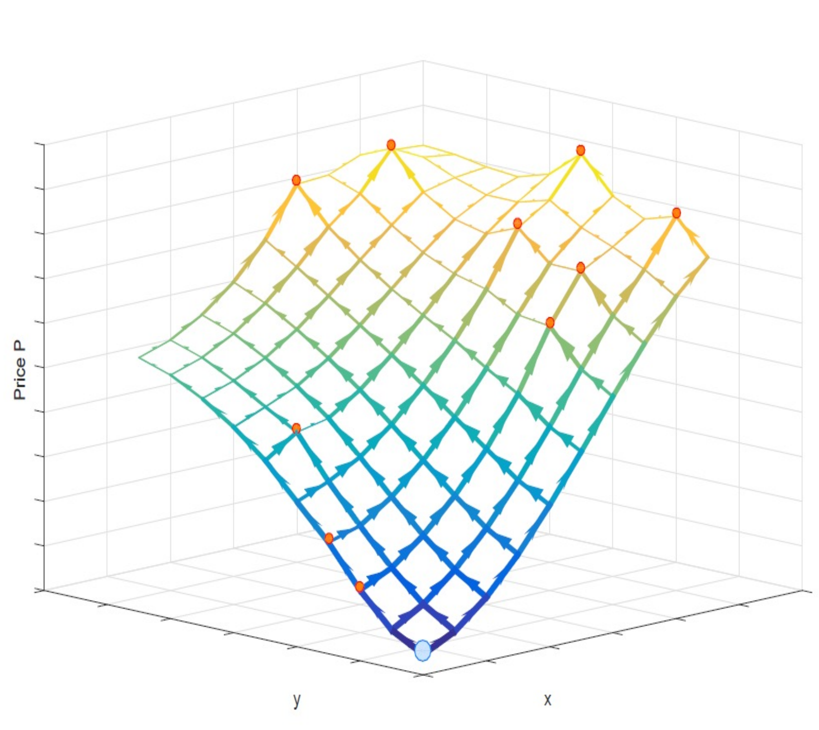
$$\boldsymbol{W} = \max_{I_{jk}} \max_{Q_{jk}^n} \max_{\{c_j, h_j, D_j^n, L_j^n, V_j^n, X_j^n\}} \sum_j w_j L_j U(c_j, h_j)$$

subject to the constraints.

  - Innermost maximization problem: standard allocation problem of choosing consumption and factor use subject to the production possibility frontier and the availability of goods in each location
  - Middle maximization problem: optimal flows subproblem over Q_{jk}^n
  - Outermost maximization problem: network design subproblem over I_{jk}.

- For given domestic absorption $\boldsymbol{D_j^n}$ and production $\boldsymbol{Y_j^n}$, the planner's problem becomes standard optimal transport problem
  - Well behaved problem, admit strong duality
  - Finding Lagrange multipliers (price) for each location-good pairs

Model Framework



- Optimal flows in a 15x15 square network with uniform infrastructure across links.
- 1 good produced at origin and consumed in 10 locations
- Price in each location indicated by z-axis
  - Solution to optimal flow problem given production, consumption and population
- Density of flows represented by thickness of links and directions by arrows
- Finding least-cost route requires information about flows, supply and demand for each good
- Optimal transport problem must be solved jointly with optimal allocation problem.

Model Framework

- **No-arbitrage condition:** The equilibrium price differential for commodity n between j and k is:

$$\frac{P_k^n}{P_j^n} \leq 1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n, \text{ if } Q_{jk}^n > 0 \qquad \text{.....(1)}$$

  - Price differential between a location and its neighbours must be less than or equal to the marginal transport cost.
  - The optimal flows follow the price gradient according to the above equation under equality. The consumption locations are local peaks of the price field as long as they do not re-ship the good.

- **Optimal Network**: As long as the upper bound is not binding, the planner's choice for I_{jk} implies

$$P_K \delta_{jk}^I \geq \sum_n P_j^n Q_{jk}^n \left(-\frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n}\right), \text{ with equality if } I_{jk} > I\_{jk} \qquad \text{.....(2)}$$

Marginal Building Cost $\geq$ Marginal Gain from Infrastructure

- Substituting the solution for Q_{jk}^n as function of the price differentials into this equation gives the optimal infrastructure I_{jk} between locations j and k as a function of prices only in each location.

Model Framework

Conditions for convexity of the planner's problem:

1. (i) Given the network $\{I_{jk}\}$, the joint optimal transport and allocation problem in the fixed (resp. mobile) labour case is a convex (resp. quasiconvex) optimization problem if $\boldsymbol{Q\tau_{jk}}(Q, I)$ is convex in Q for all j and $k \in N(j)$; and

   (ii) if in addition $\boldsymbol{Q\tau_{jk}}\ (Q, I)$ is convex in both Q and I for all j and $k \in N(j)$, then the full planner's problem including the network design problem from Definition 1 (resp. Definition 2) is a convex (resp. quasiconvex) optimization problem.

- In either the joint transport and allocation problem, or the full planner's problem, strong duality holds when labour is fixed.

Parametrization of transport costs:

Transport technology $\boldsymbol{\tau_{jk}(Q, I) = \delta_{jk}^{\tau} \frac{Q^{\beta}}{I^{\gamma}}}$, with $\beta \geq 0, \gamma \geq 0$ **....(3)**

- $\boldsymbol{Q\tau_{jk}}(Q, I)$ is convex in both arguments holds if and only if $\beta \geq \gamma$. The elasticity of per-unit transport costs to investment in infrastructure is smaller than its elasticity with respect to shipments.
- Total flows from j to k as function of prices

$$\boldsymbol{Q_{jk}^{n} = \left[\frac{1}{1+\beta} \frac{I_{jk}^{\gamma}}{\delta_{jk}^{\tau}} max \left\{\frac{P_k^n}{P_j^n} - 1, 0\right\}\right]^{1/\beta}}$$ **....(4)**

- Better infrastructure is associated with higher flows, given prices and geographic trade frictions
- Total flows decreases with congestion and increases with average price differentials.

Model Framework

Optimal infrastructure arising from unconstrained optimal-network problem (I_{jk}= 0, $\bar{I}_{jk}$ = ∞):

$$I_j^* = \left[\frac{\gamma}{P_K}\frac{\delta_{jk}^{\tau}}{\delta_{jk}^{I}}\left(\sum_n . P_j^n \left(Q_{jk}^n\right)^{.1+\beta}\right)\right]^{1/(1+\gamma)}$$

$$= \left[\frac{\gamma}{P_K \delta_{jk}^I \left(\delta_{jk}^{\tau}\right)^{1/\beta}}\left(\frac{1}{1+\beta}\sum_{n:P_k^n > P_j^n} P_j^n \left(\frac{P_k^n}{P_j^n} - 1\right)^{(1+\beta)/\beta}\right)\right]^{\beta/(\beta-\gamma)}$$

- Given the prices at origin, the optimal infrastructure increases with gross flows.
- Given gross flows, the optimal infrastructure increases with prices at origin.
- Optimal investment increases with δ_{jk}^{τ} i.e. optimal investment offsets geographic trade frictions.
- Optimal investment decreases with δ_{jk}^{I} i.e. less investment when infrastructure costlier to build

2. **Optimal Network in Log-linear case**: When the transport technology is given by (3), the full planner's problem is a convex (resp. quasiconvex) optimization problem if $\beta \geq \gamma$.* The optimal infrastructure is given by I_{jk}= min[max(I_{jk}^*, I_{jk}), $\bar{I}_{jk}$] implying that, in the absence of a pre-existing network ($I\_{jk} = 0$), then $I_{jk} = 0 \Leftrightarrow P_k^n = P_j^n$ for all n.

Model Framework

Non-Convexity: Increasing Returns to Transport

- In the absence of a pre-existing network and transport technology given by (3) with $\beta \geq 0, \gamma \geq 0$ and satisfies $\gamma > \beta$, non-convexity arise as the transport technology features economies of scale, and if there is a unique commodity produced in a single location, the optimal transport network is a tree.
- In cases of multiple goods and multiple production locations with $\gamma > \beta$, the optimal network is sparser and concentrated on fewer links relative to cases with $\gamma \leq \beta$.

Decentralized Competitive Equilibrium

- For a given network $\{\boldsymbol{I_{jk}}\}$, planner's optimal allocation and optimal transport correspond to a decentralized competitive equilibrium of a standard neoclassical economy where consumers maximize utility, producers maximize profits and goods and factor markets clear. Only less standard feature is existence of a transport sector with congestion.
- The optimal route maximizes the per-unit profits:

$$\pi_{od}^{n} = \max_{r=()} p_d^n - p_o^n - \sum_{k=0}^{\rho-1} p_{jk}^n \tau_{jk\,jk+1}^n - \sum_{k=0}^{\rho-1} p_{jk}^n t_{jk\,jk+1}^n$$

Selling price Sourcing cost Transport cost Taxes (tolls)

Model Framework

Decentralized Competitive Equilibrium

Definitions for Planner's problem

- Without labour mobility
- With labour mobility

Definition 3. *The decentralized equilibrium without labor mobility consists of quantities* $c_j, h_j, D_j, D_j^n, L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n, \{Q_{jk}^n\}_{k\in\mathcal{N}(j)}$, *goods prices* $\{p_j^n\}_n, p_j^D, p_j^H$ *and factor prices* $w_j, \{r_j^m\}_m$ *in each location* j*such that:*

(i)(a) consumers optimize:

$$\{c_j, h_j\} = \arg\max_{\hat{c}_j, \hat{h}_j} U\left(\hat{c}_j, \hat{h}_j\right)$$

$$p_j^D \hat{c}_j + p_j^H \hat{h}_j = e_j \equiv w_j + t_j,$$

where e_j *are expenditures per worker in* j *and where* p_j^D *is the price index associated with* $D_j\left(D_j^1, ..., D_j^N\right)$ *at prices* $\{p_j^n\}_n$ *and* t_j *is a transfer per worker located in* j. *The set of transfers satisfy*

$$\sum_j t_j L_j = \Pi$$

where Π *adds up the aggregate returns to the portfolio of fixed factors and the government tax revenue,*

$$\Pi = \sum_j p_j^H H_j + \sum_j \sum_m r_j^m V_j^m + \sum_j \sum_{k\in\mathcal{N}(j)} \sum_n t_{jk}^n p_k^n Q_{jk}^n;$$

(i)(b) firms optimize:

$$\{L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\} = \underset{\hat{L}_j^n, \hat{\mathbf{V}}_j^n, \hat{\mathbf{X}}_j^n}{argmax}\; p_j^n F_j^n\left(\hat{L}_j^n, \hat{\mathbf{V}}_j^n, \hat{\mathbf{X}}_j^n\right) - w_j \hat{L}_j^n - \sum_m r_j^m \hat{V}_j^{mn};$$

(i)(c) the transport companies optimize,

$$\pi_{od}^n = \max_{r=(j_0,\dots,j_\rho)\in\mathcal{R}_{od}} p_d^n - p_o^n - \sum_{k=0}^{\rho-1}\left(\chi P_{j_k}^D m^n \tau_{j_k j_{k+1}} + (1-\chi) P_{j_k}^n \tau_{j_k j_{k+1}}^n\right) - \sum_{k=0}^{\rho-1} p_{j_{k+1}} t_{j_k j_{k+1}}^n,$$

for all $(o,d) \in \mathcal{J}^2$, *where* $\mathcal{R}_{od} = \left\{(j_0, \dots, j_\rho) \in \mathcal{J}^{\rho+1}, \rho \in \mathbb{N} \mid j_0 = o,\, j_\rho = d,\, j_{k+1} \in \mathcal{N}(j_k) \text{ for all } 0 \le k < \rho\right\}$ *is the set of routes from* o *to* d,*and there is free entry to delivering products from every source to every destination:* $\pi_{od}^n \le 0$ *for all* $(o,d) \in \mathcal{J}^2$, $=$ *if good* n *is shipped from* o *to* d.

(i)(d) producers of final commodities optimize:

$$\{D_j^n\} = \underset{\hat{D}_j^n}{argmax}\; D_j\left(\left\{\hat{D}_j^n\right\}\right) - \sum_j p_j^n \hat{D}_j^n;$$

as well as the market-clearing and non-negativity constraints (i), (ii), (iv), and (v) from Definition 1.

If, in addition, labor is mobile, then the decentralized equilibrium also consists of utility u *and employment* $\{L_j\}$ *such that*

$$u = U_j\left(c_j, h_j\right)$$

whenever $L_j > 0$, *and the labor market clearing condition (vii) from Definition 2 holds.*

Model Framework

Welfare Theorems:

- If the tax on shipments of product n from j to k is $\boldsymbol{t_{jk}^{n} = \varepsilon_{Q,jkn}^{\tau} \tau_{jk}^{n}}$ where $\varepsilon_{Q,jkn}^{\tau} = \frac{\partial \log \tau_{jk}^{n}}{\partial \log Q_{jk}^{n}}$ then,
  (i) if labour is immobile, the competitive allocation coincides with the planner's problem under specific planner's weights w_j. Conversely, the planner's allocation can be implemented by a market allocation with specific transfers t_j; and
  (ii) if labour is mobile, the competitive allocation coincides with the planner's problem if and only if all workers own an equal share of fixed factors and tax revenue regardless of their location, i.e., $\boldsymbol{t_j = \frac{\Pi}{L}}$.
- In either case, the price of good n in location j, p_j^n equals the multiplier on the balanced-flows constraint in the planner's allocation, P_j^n.
- Optimal allocation can be equivalently implemented by per-unit toll $\boldsymbol{\theta_{jk}^{n} = p_{jk}^{n} \varepsilon_{Q,jkn}^{\tau} \tau_{jk}^{n}}$
- If the global convexity condition 1 is satisfied and the toll θ_{jk}^{n} is consistent with the optimal Pigouvian tax ($\theta_{jk}^{n} = P_j^n \varepsilon_{Q,jkn}^{\tau} \tau_{jk}^{n}$), then the decentralized infrastructure choice implements the optimal network investment.

Model Framework

Extension of model framework to cases where congestion occurs across goods

- Per-unit cost τ_{jk}^{n} is denominated in units of the bundle of traded goods aggregated through D_j $(\cdot)$ rather that in units of the good itself. Assuming that transporting each unit of good n from j to $k \in N(n)$ requires $\tau_{jk}^{n} = m^{n}\tau_{jk}(Q_{jk}, I_{jk})$ units of the traded goods bundle, Number of units of the traded goods bundle D_j used to transport goods from j to its neighbours is $T_j = \sum_{k \in N(j)} \tau_{jk}(Q_{jk} I_{jk}) Q_{jk}$
- After properly adjusting the resource constraints in the definition of the planner's problem, the convexity of the full planner's problem is preserved under the same conditions stated in Definition 1 of convexity.

Extension of model framework to cases of Inefficient Market Allocation

- Several externalities lead to inefficient market allocation
  - Production technology is $Y_j^n = F_j^n(L_j^n, V_j^n, X_j^n; L_j)$ where the spillovers from the total number of workers L_j on output Y_j^n is not internalized in the market allocation.
  - Consumption of amenities entering through utility, $U(c_j, h_j; L_j)$
  - Pigouvian taxes t_{jk}^{n} doesn't correct the congestion externality in shipping
- The general convexity of the problem corresponding to part (ii) of Definition 1 can't be established

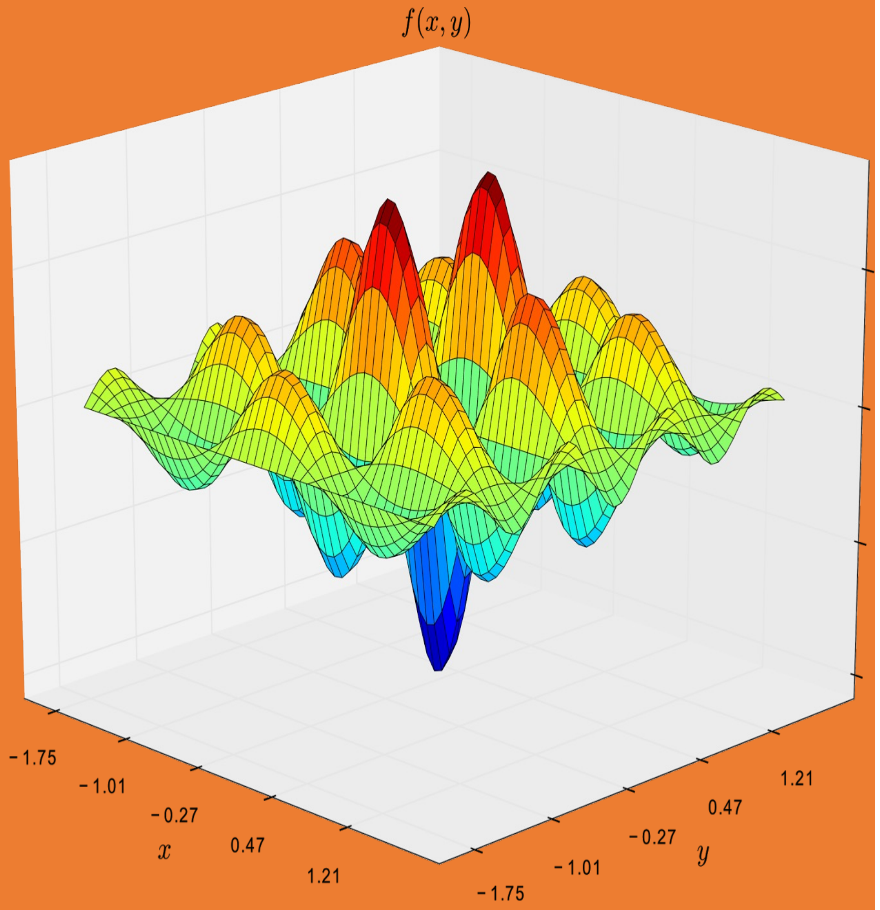
Model Algorithm

Convex Cases (Definition 1)

- The full planner's problem is a convex optimization problem
- KKT conditions are necessary and sufficient
- 1st order conditions is a system of non-linear equations with many unknowns
- Gradient-descent based algorithms used to make optimization problem tractable
  - Feed the numerical solver with the primal problem $\sup_{x} \inf_{\lambda \geq 0} \mathcal{L}(x, \lambda)$
  - Solve the dual problem by inverting the order of optimization

    $$\inf_{\lambda \geq 0} \sup_{x} \mathcal{L}(x, \lambda) = \inf_{\lambda \geq 0} \sup_{x} \mathcal{L}(x(\lambda), \lambda)$$
- Convexity of problem ensures that the dual coincides with the primal i.e. strong duality holds

Model Algorithm

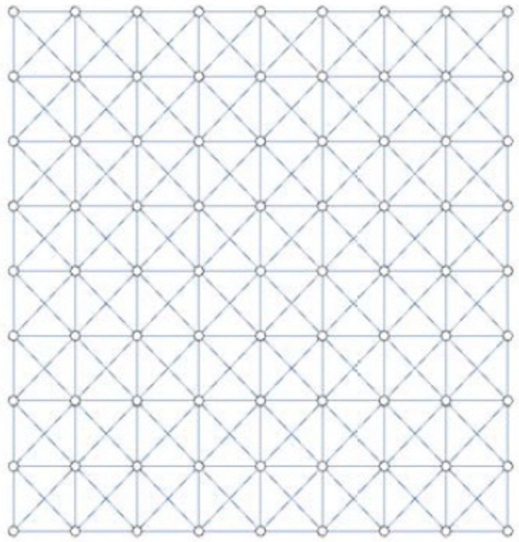


Non-Convex Cases

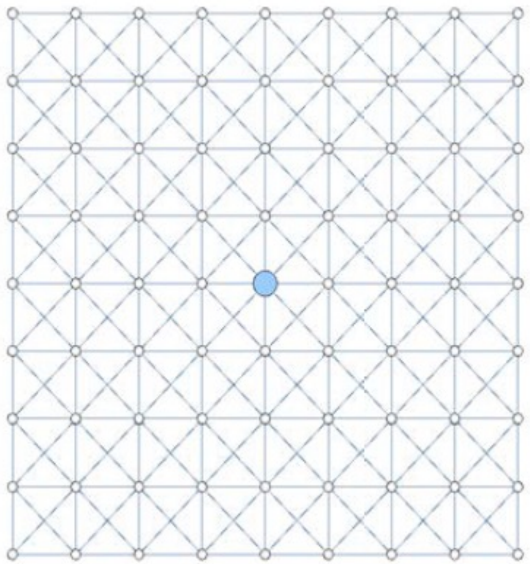
- Combine the primal and dual approaches to solve for the joint neoclassical allocation and optimal transport problems with an iterative procedure over the infrastructure investments
    1. For i :=1, guess some initial level of infrastructure $\left\{I_{jk}^{(1)}\right\}$ that satisfies the network building constraint.
    2. Given the network $\left\{I_{jk}^{(1)}\right\}$, solve for $\{c_j, D_d^n, Q_{jk}^n, L_j^n\}$ using a duality approach.
    3. Given the flows $\{Q_{jk}^n\}$ and prices P_j^n , make a new guess $\boldsymbol{I_{jk}^{(i+1)}} = \left[\frac{\gamma\ \delta_{jk}^{\tau}}{P_K \delta_{jk}^{I}} \left(P_j^D Q_{jk}^{1+\beta}\right)\right]^{\frac{1}{1+\gamma}}$ for x=1 and $\boldsymbol{I_{jk}^{(i+1)}} = \left[\frac{\gamma\ \delta_{jk}^{\tau}}{P_K \delta_{jk}^{I}} \left(\sum_n P_j^n \left(Q_{jk}^n\right)^{1+\beta}\right)\right]^{\frac{1}{1+\gamma}}$ for x=0. and set P_K s.t. $\sum \delta_{jk}^I I_{jk}^{(i+1)} = K$
    4. If $\sum_i \left|\boldsymbol{I_{jk}^{(i+1)}} - \boldsymbol{I_{jk}^{(i)}}\right| \leq \boldsymbol{\varepsilon}$, then potential candidate for a local optimum achieved, likely to be a local extremum. If not, set I := i+1 and go back to 2.
- Use Simulated Annealing to guarantee convergence to a global maximum.
    1. For i :=1, set the initial network $\{I_{jk}^{(1)}\}$ to a local optimum as achieved, and compute its welfare $v^{(1)}$. Set the initial temperature T of the system to some number.
    2. Draw a new candidate network $\{\hat{I}_{jk}\}$ by perturbing $\left\{I_{jk}^{(i)}\right\}$. Compute the corresponding optimal allocation and transport $\{c_j, D_d^n, Q_{jk}^n, L_j^n\}$. Compute associated welfare $\hat{v}$.
    3. Accept the new network i.e. set $I_{jk}^{(i+1)} = \hat{v}$ and $v^{i+1} = \hat{v}$ with probability min[exp(($\hat{v}$ - v^1)/T), 1], if not keep the same network $\left\{I_{jk}^{(i+1)}\right\} = \left\{I_{jk}^{(i)}\right\}$ and $v^{i+1} = v^i$
    4. Stop when $T < T_{min}$. Otherwise set i := i+1 and $T := \rho_T T$ and return to 2.

Illustrative Examples

(a) Population



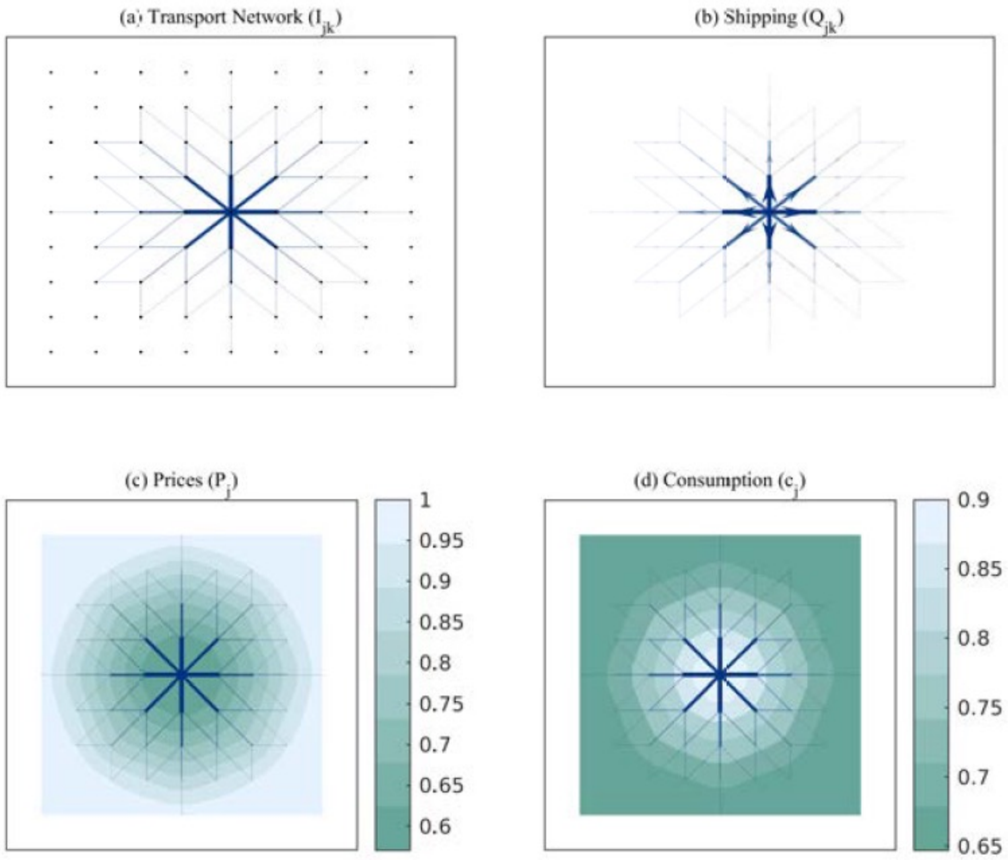
(b) Productivity

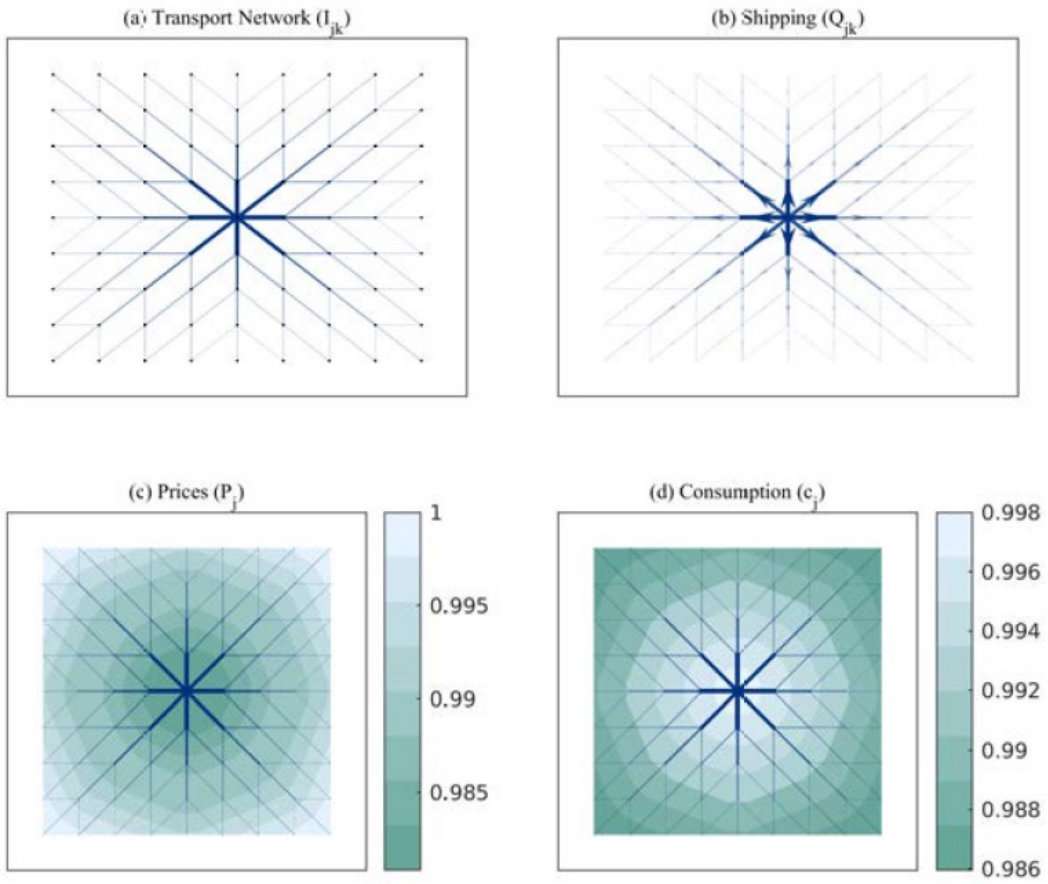


- Each circle represents a location.
- Links represent the underlying network upon which the transport network may be built.
- Population and housing are uniform across space, normalized to 1.

- The size of the circles represent the productivity of each location
- Productivity at center assumed 10 times that at any other location.

Illustrative Examples





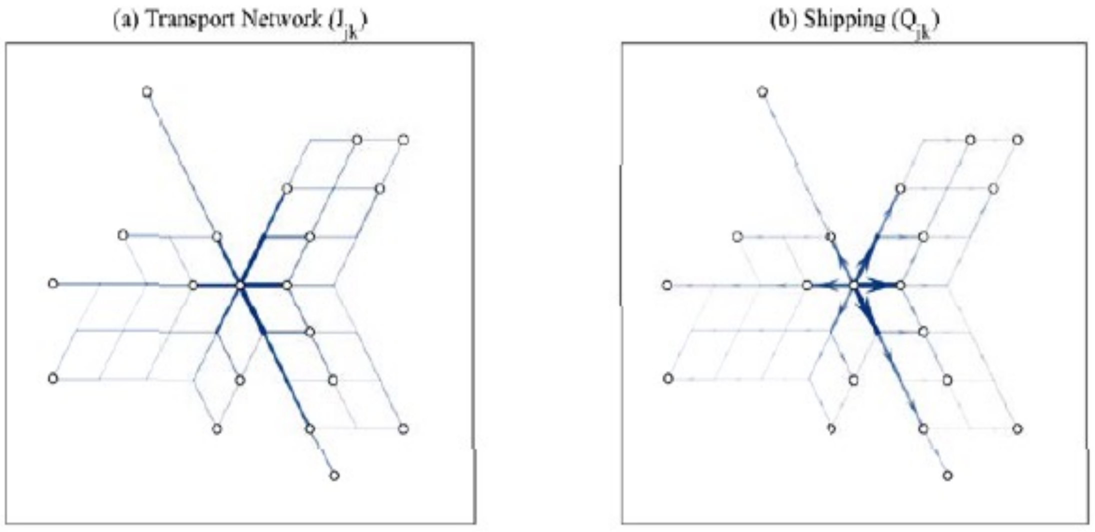
Optimal Transport Network for K=100

Optimal Transport Network for K=1

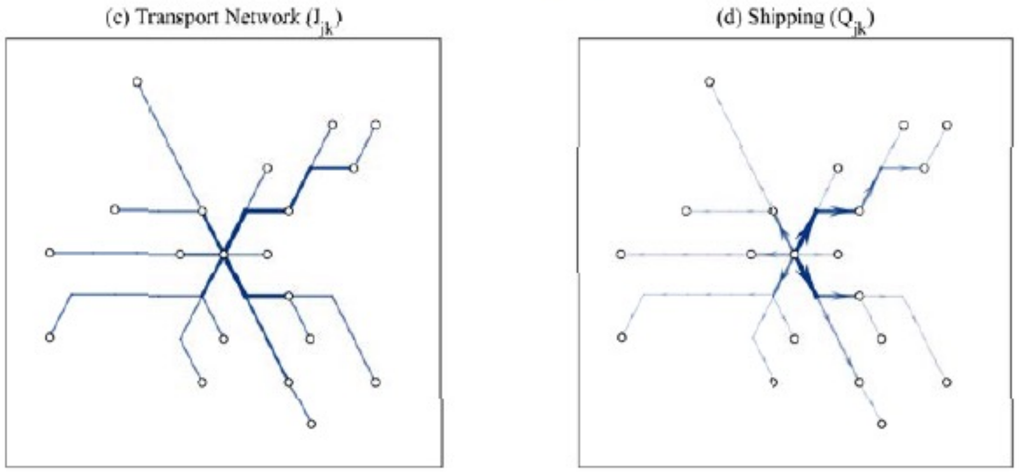
Illustrative Examples

Optimal Transport Network with Randomly located cities

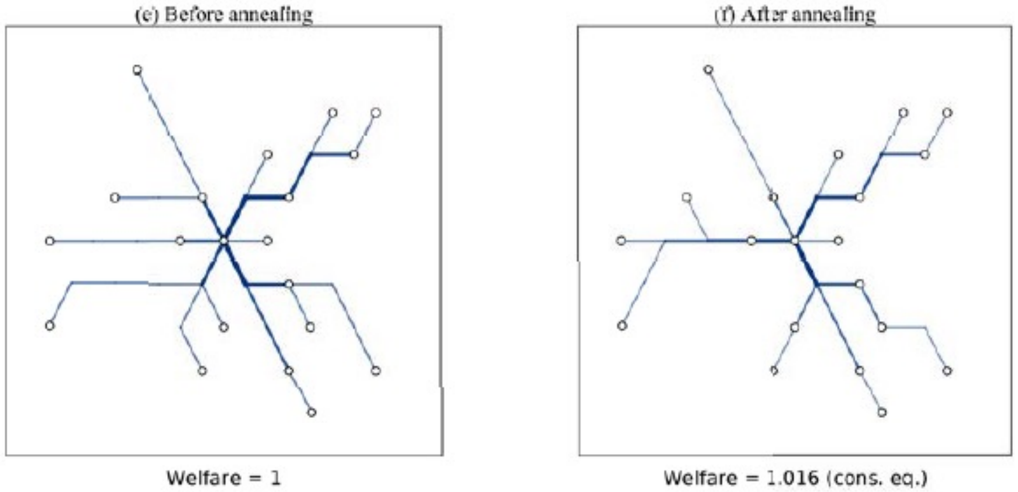
(a) Convex Case: $\gamma = \beta = 1$



(b) Non-Convex Case: $\gamma = 2 > \beta = 1$

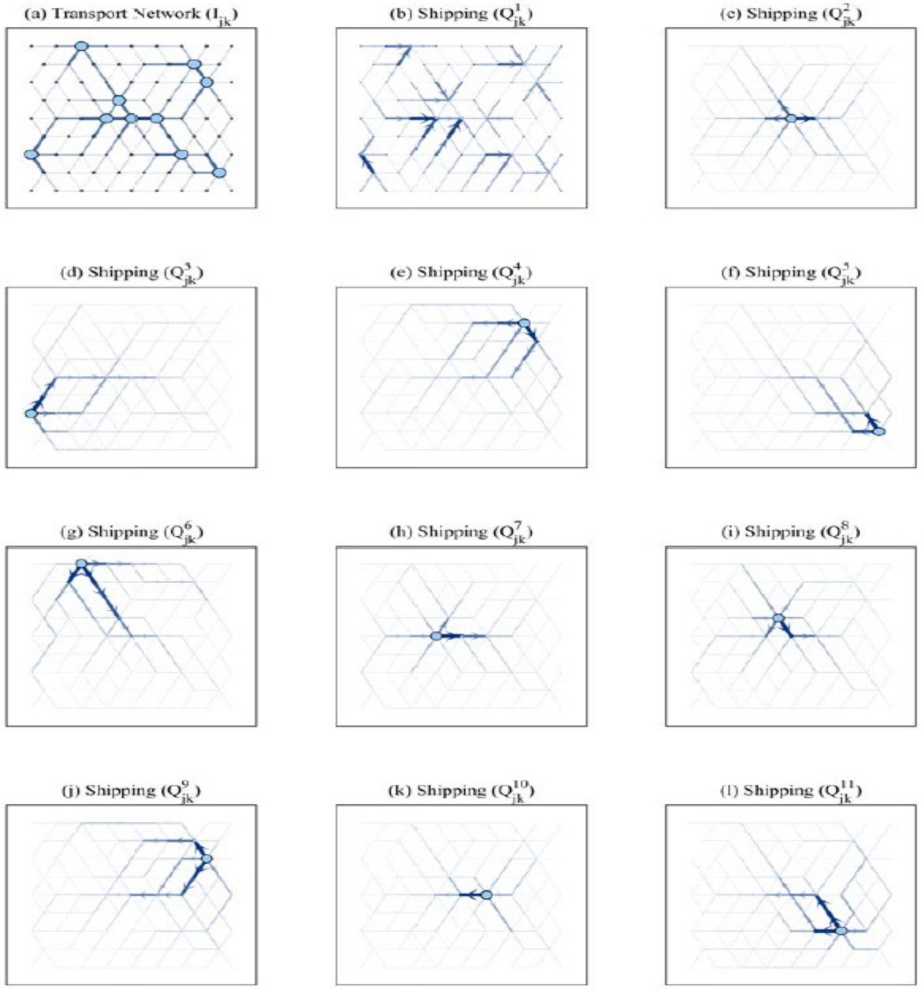


(c) Optimal Network Before and After Annealing Refinement in Non-Convex Case



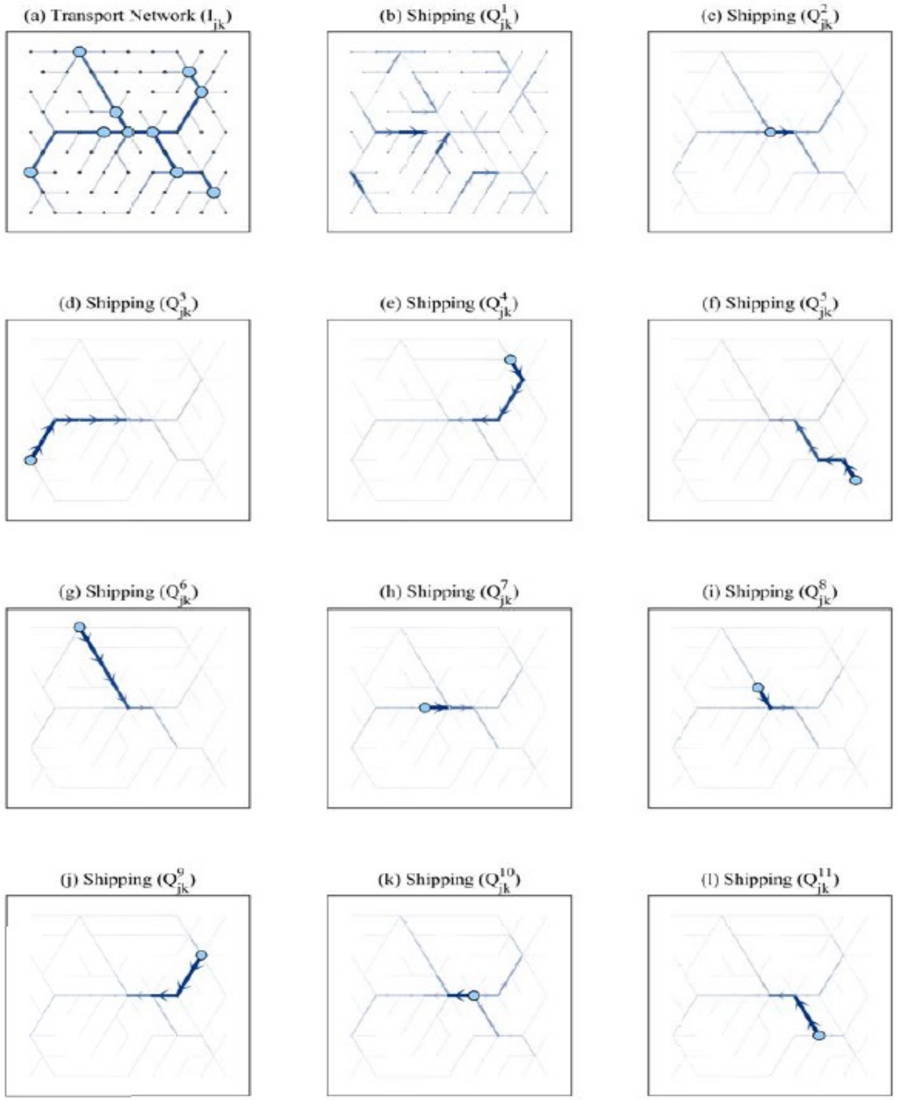
Illustrative Examples

Optimal Transport Network with 10+1 Goods

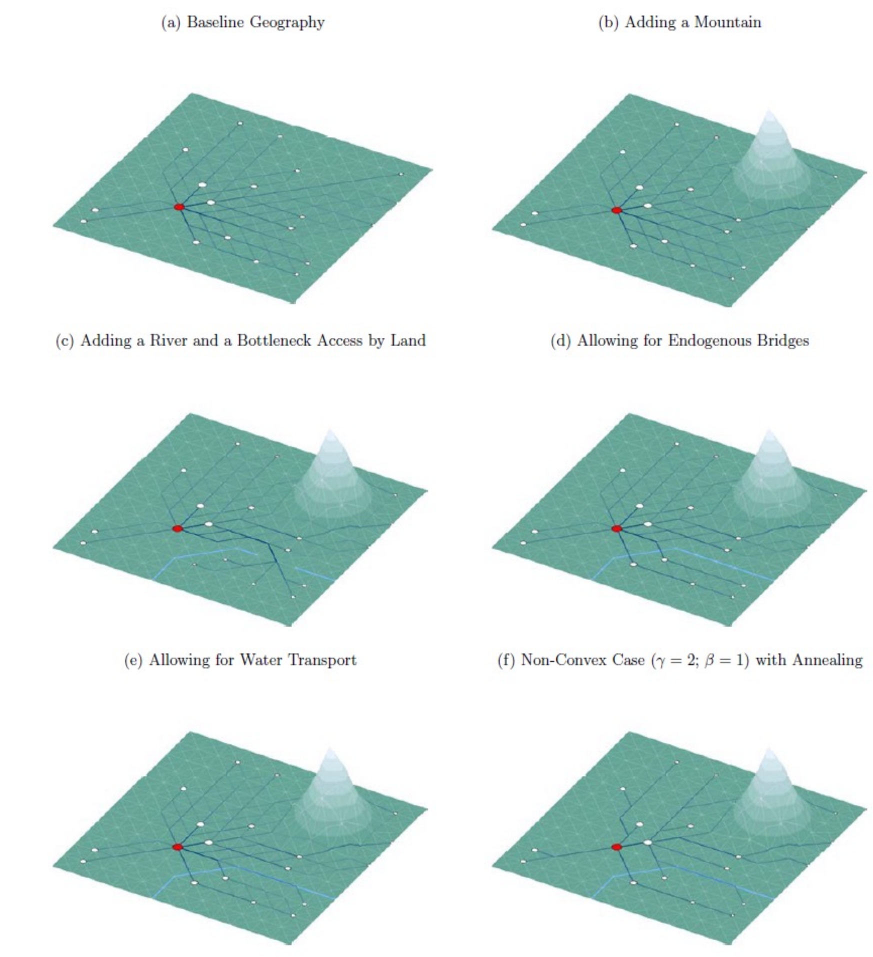


Convex Case with Labour mobility

Non-Convex Case with Labour mobility



Illustrative Examples



Optimal Transport Network under Alternative Building Costs

- Thickness and colour of the segments reflects the level of infrastructure built on a given link.
- Thicker and darker colours represent more infrastructure and quantities.
- Circles represent the 20 cities randomly allocated across spaces. The larger red circle represents the city with the highest productivity.
- Different panels vary in the parametrization of the cost of building infrastructure.
- Panel (a): Euclidean distance
- Panel (b): A mountain added
- Panel (c): A river with a natural land crossing added
- Panel (d): No land crossing. Bridge construction over river added with increasing returns to network building
- Panel (e): Bridge construction over river added

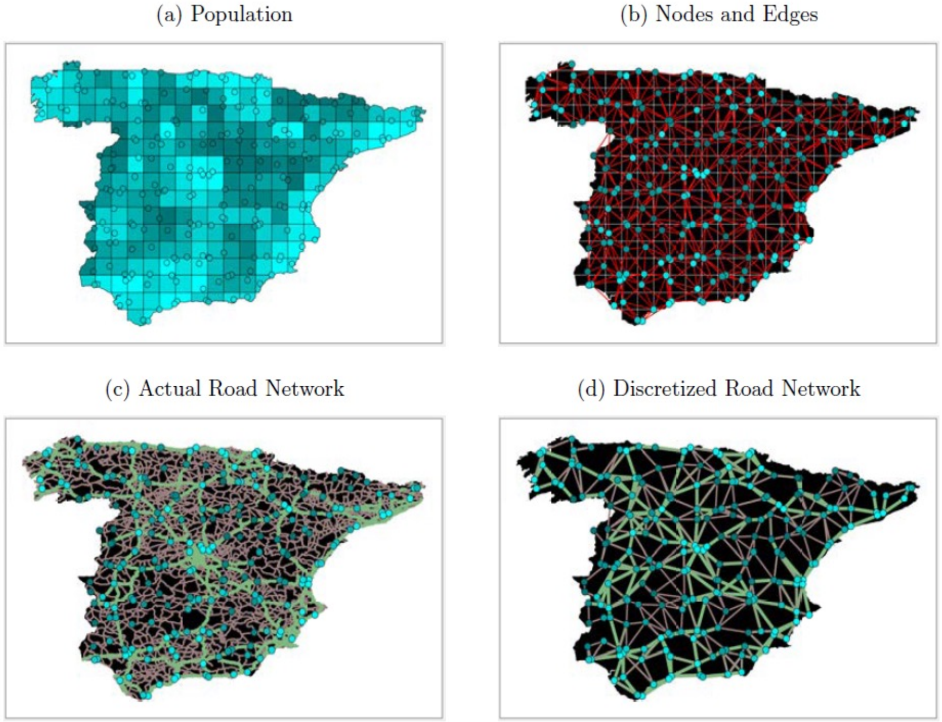
Application of Framework to European Road Networks

Assumptions:

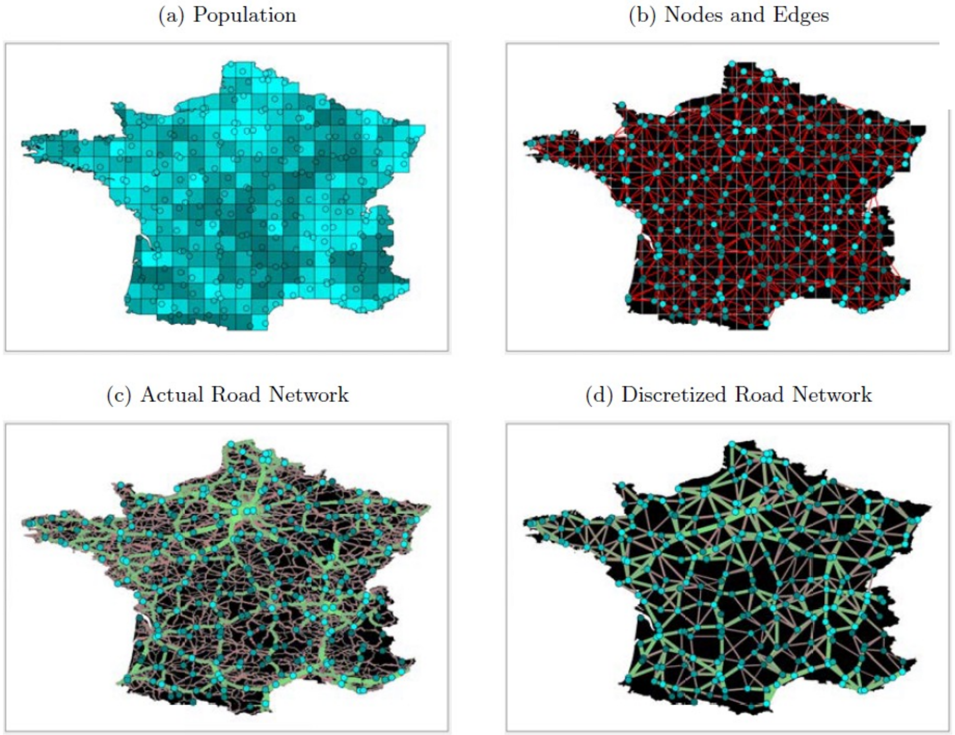
- Individual utility over traded/non-traded goods to be Cobb-Douglas $\boldsymbol{U = c^{\alpha} h^{1-\alpha}}$
- Aggregator of traded goods to be CES with $c_j = \left[\sum_{n=1}^{N} (\boldsymbol{c_j^n})^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$
- Labour is the only factor of production
- Production technologies to be linear
- $\boldsymbol{\sigma} = 5, \boldsymbol{\alpha} = 0.4$
- Congestion parameter and the returns to infrastructure parameter should be set to elasticities of total trade costs with respect to trade flows and infrastructure. Since such elasticities are not readily available, we use the impact of shipping time on trade costs
- Shipping speed is a loglinear function of the number of vehicles and road lane kilometres;
- Number of vehicles is a linear function of the quantity shipped.

- Under these assumptions, we can calibrate $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ to match empirical relationship between speed, roads and vehicles estimated. Their estimates imply $\boldsymbol{\beta} = 0.13$ and $\boldsymbol{\gamma} = 0.10$ suggesting decreasing returns to scale.
- We use these parameters as benchmark, and also implement the analysis in a case with increasing returns. Specifically, considering a higher value of $\boldsymbol{\gamma}$ such that the ratio between $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ is the mirroring case ($\boldsymbol{\beta}$=0.13 and $\boldsymbol{\gamma} = 0.169$)

Application of Framework to European Road Networks

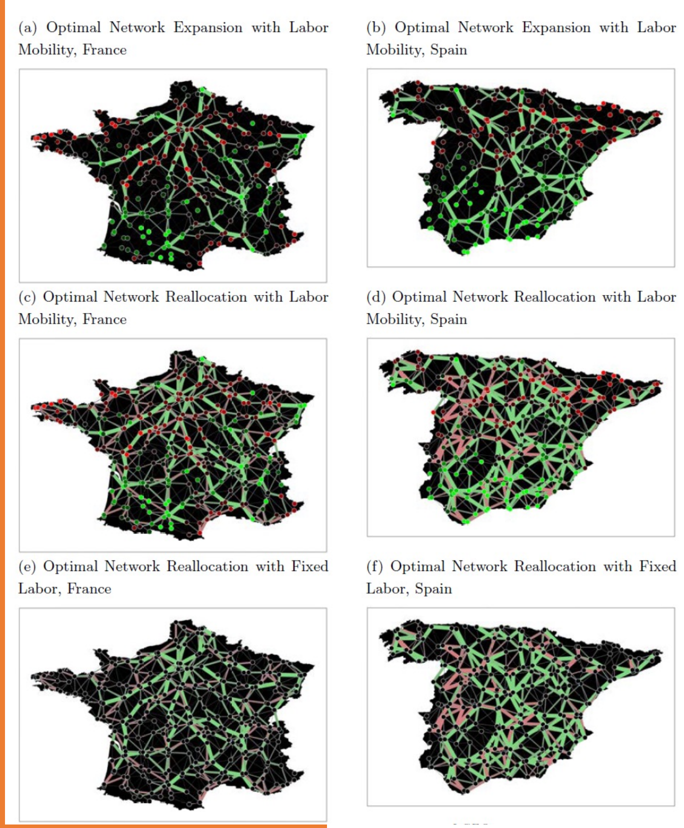


Discretisation of Spanish Road Network



Discretisation of French Road Network

Application of Framework to European Road Networks



Counterfactuals simulation

1. Optimal expansion: Compute the aggregate gains from an optimal expansion 50% of the observed road network (total resources K) within each country constraining the planner to build on top of the existing network, $I\_{jk} = I_{jk}^{obs}$.
   - More policy relevant
2. Optimal reallocation: Compute the losses due to misallocation of current roads within each country without constraining the planner to build on top of the existing network, I_{jk}^{obs} s. t. I_{jk}=0.

- All counterfactuals use the geographic measure of building costs, $\boldsymbol{\delta^{I,GEO}}$, and the benchmark parametrization of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.
- Width and brightness of each link is proportional to the difference between the optimal counterfactual network and the observed network, $I_{jk}^{*} - I_{jk}^{obs}$ for each link $jk \in E$.
- Comparison between (c)-(d) and (e)-(f) reveals **labour mobility does not fundamentally affect the optimal infrastructure investments.**

Conclusions

- Optimal road investments are directed to locations with initially lower levels of infrastructure, reflecting decreasing returns to infrastructure at the link level.
  - Under GEO-based measure of building costs, the investments are also more intensely directed to locations with initially higher levels of population and income per worker.
  - Under FOC-based measure of building costs, the observed allocation of roads is efficient and therefore the welfare impact of infrastructure is equalized across links regardless of the fundamentals reflected in income and population.
- Since the model implies a complex mapping from the fundamentals to the investments, these observable outcomes guide only a fraction of the optimal investment decisions (R^2 ~20-30%).
- There is lack of significant correlation between infrastructure growth and population growth, which implies that when the optimal investment plan is implemented, growth in a location depends on investments in other locations in potentially complex ways. If we randomly improve individual links, population growth does appear correlated with infrastructure growth.
- Impact of initial income on population growth in the optimal investment plan operates through the level of consumption reflecting that the goal of the optimal investments is to reduce variation in the marginal utility of consumption of traded commodities across locations.
- The optimal investment in infrastructure reduces spatial inequalities, although different assumptions on building costs imply different ways of achieving this goal by changing the optimal placement of infrastructure.

Conclusions

- A general framework developed to study optimal transport networks in spatial equilibrium.
- Provides conditions such that the full planner's problem, involving the optimal flow of goods as well as the general-equilibrium and network-design problems, is globally convex and numerically tractable
- Gains from road expansion and real consumption losses from misallocation. Suggests a way for policy planners to evaluate the welfare utilities for policy formulations.
- Role of regional characteristics such as institutional quality in infrastructure investment

Limitations

- Assumes that the economy is in a state of general equilibrium which may not be realistic in all cases.
- No conclusive relations between populations and infrastructure growth due to over-generalised model framework.
- Model framework developed for only 10 commodities in trade within an economy. Increase in number of commodities make the model highly difficult to solve because of heuristic approach of simulated annealing.
- No account of the impact of factors, e.g. climate change, on transportation policy

Future Research

- Trade model development for developing economies in a neoclassical framework
- Effect of a global planner in economies having strong agglomeration.
- Better parametrization to find a conclusive relation between population and infrastructure growth.
- Construct instruments for policy planners for investments in transport infrastructure as function of observable regional characteristics.
- Study political economy issues associated with infrastructure, such as spatial competition among planning authorities.

Thank you for listening !!!