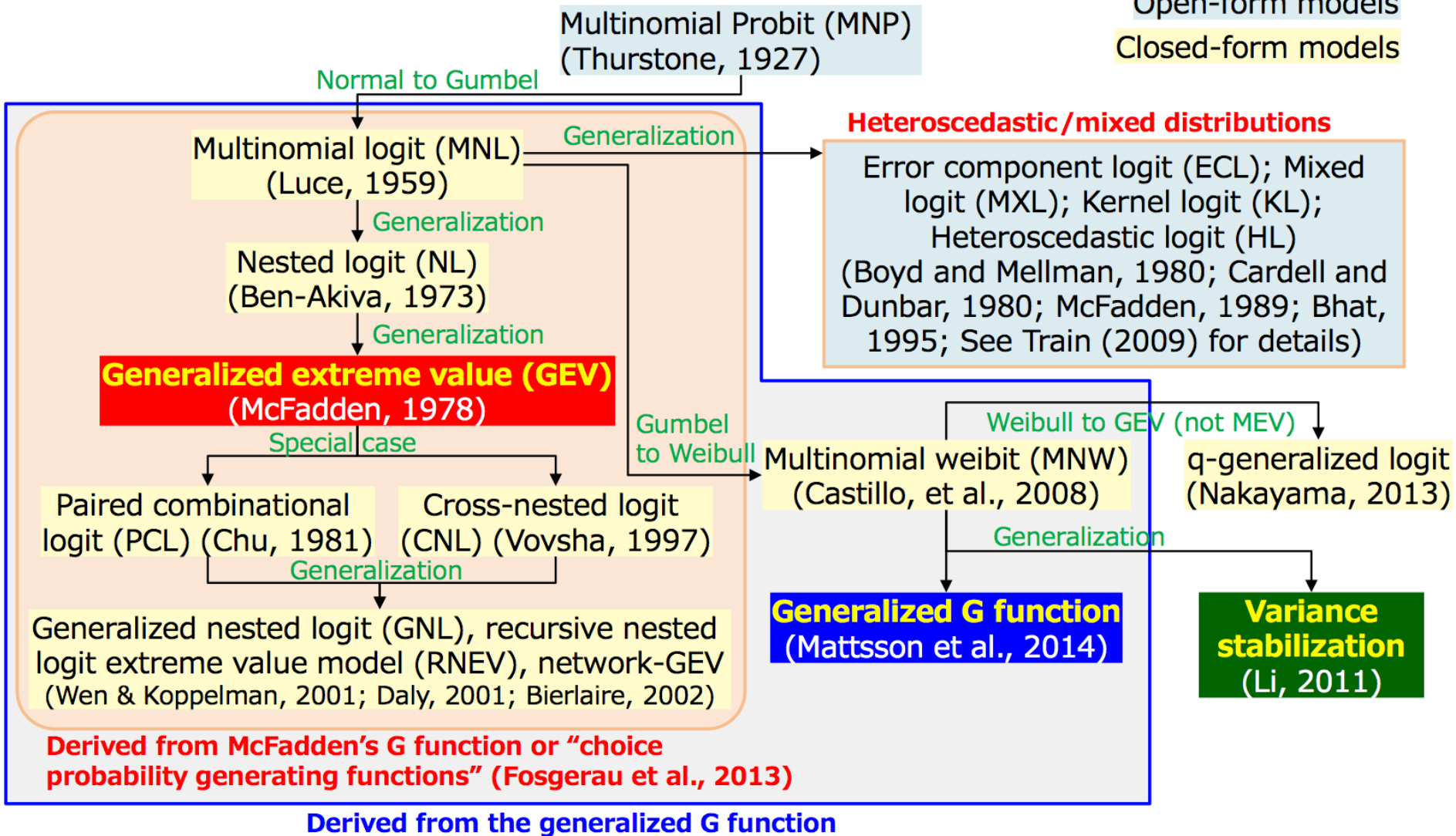


# **Open-form discrete choice models**

# Genealogy of DCMs (again)

Open-form models  
Closed-form models



## GEV model (Closed-form)

Multinomial Logit (MNL)

$$P(i) = \frac{\exp(\mu V_i)}{\sum_{j \in C} \exp(\mu V_j)}$$

- Luce(1959), McFadden(1974)
- Not consider correlation of choice alternatives' (IIA)
- Easy and fast estimation
- High operability  
(easy evaluation for new additional choice alternative  $\Rightarrow$  benefit of IIA)

## Non-GEV model (Open-form)

Multinomial Probit (MNP)

$$P(i) = \int_{\varepsilon_1=-\infty}^{\varepsilon_i+V_i-\varepsilon_1} \cdots \int_{\varepsilon_i=-\infty}^{\infty} \cdots \int_{\varepsilon_j=-\infty}^{\varepsilon_i+V_i-\varepsilon_j} \phi(\varepsilon) d\varepsilon_j \cdots d\varepsilon_1$$
$$\phi(\varepsilon) = \frac{1}{(\sqrt{2\pi})^J |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \varepsilon \Sigma^{-1} \varepsilon'\right)$$

- Thurstone(1927)
- Consider correlation of choice alternates' based on Variance-Covariance matrix
- Hard and slow estimation  
(need calculation of **multi-dimensional interrelation** depend on N of alternatives')

Non-GEV model has high power of expression, however parameter estimation cost is high.

# Structured Covariance MNP (1)

3

Multinomial Probit with Structured Covariance for Route Choice Behavior, Transportation Research Part B, Vol.31, No.3, pp195-207, 1997.



Prof. Morichi



Prof. Yai



Prof. Iwakura

- Proposed new probit type railway route choice model considering overlapping problem. (1993, 1998)
- This model applied to practical demand forecasting in real Tokyo network, and it used for decision making of railway policy toward 2015. (2000)



Pergamon

Transp Res.-B, Vol. 31, No. 3, pp 195-207, 1997  
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## MULTINOMIAL PROBIT WITH STRUCTURED COVARIANCE FOR ROUTE CHOICE BEHAVIOR

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(Received 8 September 1994; in revised form 19 June 1996)

**Abstract**—We propose another version of the multinomial probit model with a structured covariance matrix to represent any overlapped relation between route alternatives. The fundamental ideas of the model were presented in Yai *et al.* (1993) and Yai and Iwakura (1994). The assumptions introduced in the model may be more realistic for route choice behaviors on a dense network than the strict assumption of the independent alternative property of the multinomial logit model. As the nested logit model assumes an identical dispersion parameter between two modeling levels for all trip makers, the model has difficulty in expressing individual choice-tree structures. To improve the applicability of the multinomial probit model to route choice behaviors, we introduce a function which represents an overlapped relation between pairs of alternatives and propose a multinomial probit model in which the structured covariance matrix uses the function in order to consider the individual choice-tree structures in the matrix and the estimability of the new alternative's covariances. After examining the applicability of the multinomial probit model using empirical route choice data in a Tokyo metropolitan region, we also propose a method for evaluating consumer benefits on complicated networks based on the multinomial probit model. © 1997 Elsevier Science Ltd

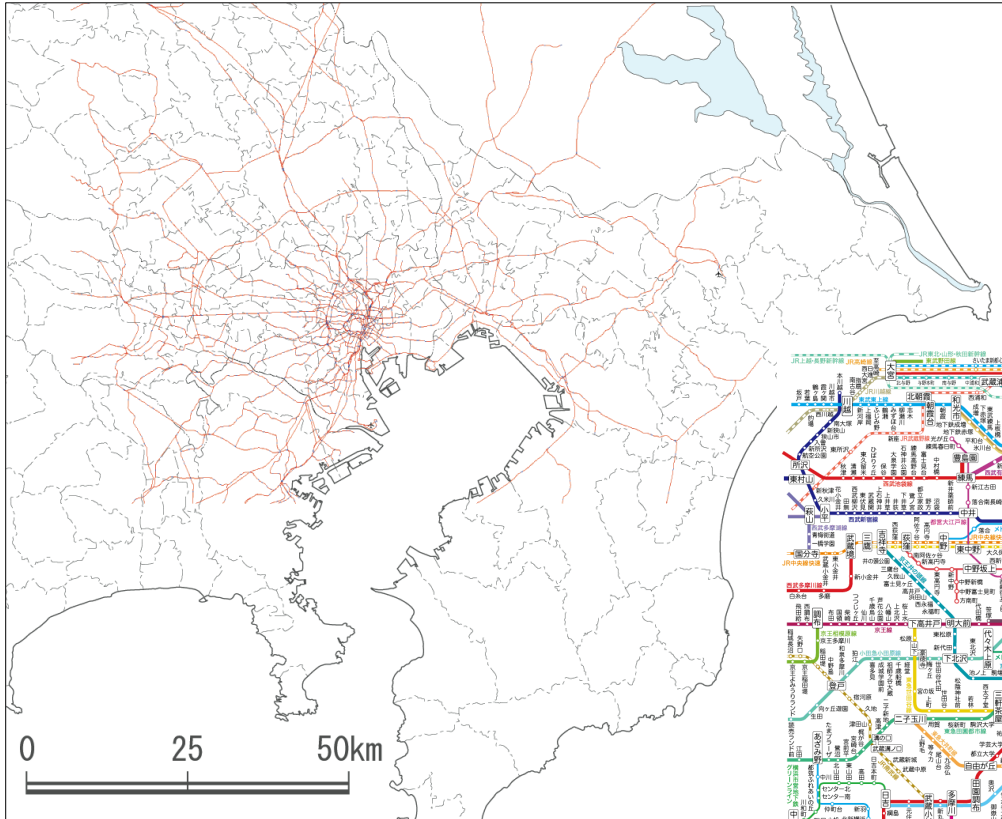
### 1. INTRODUCTION

The applications of the multinomial probit model have not been adequately successful in spite of its advantages in flexibility of the model form. Certainly, the complexity of the computational process has deterred its use, compared to the wide applications of the multinomial logit models. Early advances in the estimation method of the multinomial probit model were achieved before the early 80s, by Daganzo (1977), Lerman and Manski (1981), Daganzo and Sheffi (1982) and Sheffi *et al.* (1982). Their work discussed alternative methods for estimating the covariance matrix simultaneously with utility function parameters. No accurate method was found during these earlier advances and thus the multinomial probit model was not widely applied (Horowitz *et al.*, 1982; Horowitz, 1991). In the 1980s, most discrete choice models were calibrated by the multinomial logit model or expansion forms of the multinomial logit such as the nested logit model. Although most results were satisfactory in representing travel behaviors of modal choices, several behaviors which do not satisfy the assumptions of the multinomial logit model exist. Most probably, the cause of such behaviors is the interdependency of choice alternatives.

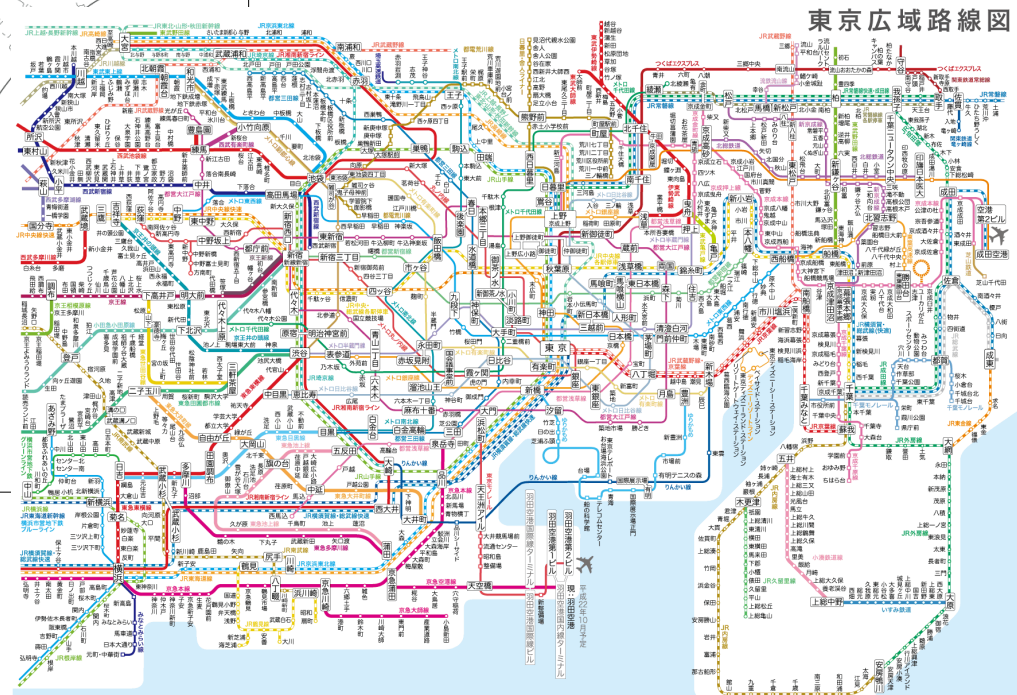
Recently, there have been advances in multinomial probit estimation (McFadden, 1989; Pakes and Pollard, 1989; Bunch, 1991; Bolduc and Ben-Akiva, 1991; Bolduc, 1992; Geweke *et al.*, 1994). The method of simulated moments proposed by McFadden seems to encourage multinomial probit applications because of its computational efficiency in seeking model parameters. Bolduc focused on the estimation of the multinomial probit model with a large choice set using auto-regressive errors with distance related functions among alternatives for simplifying its covariance matrix. Bunch simplified the multinomial probit model's covariance matrix with his transformation method which lessens the estimation problem. Geweke *et al.* compared several

# Structured Covariance MNP (2)

Tokyo Metropolitan has highly dense railway network !  
⇒ route overlapping problem



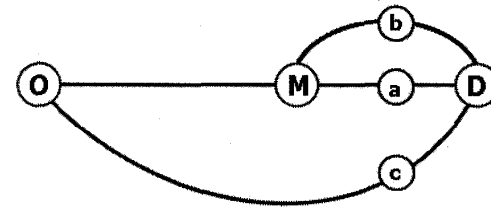
Railway line : about 130  
Station : about 1800  
Passengers : 40million/day  
Cong. rate: max over 200%



©2010 国鉄グループ  
※上記路線図は、個人・法人問わず、複製・改作・再配布・転載を禁じます。

In the overlap network that has **correlation** between routes, Logit model is susceptible to **error** by IIA property.

Overlap = correlation



Probit is better ?

- Difficult to setting covariance matrix for each OD pair  
⇒ structured covariance by divide into two error
- Difficult to parameter estimation. (multi-dimensional Integral)  
⇒ reduce computational time using simulation methods

$$U_i = V_i + \varepsilon_i$$

$$\varepsilon_i = \varepsilon_i^{Length} + \varepsilon_i^{Route}$$

Error of depend on route length

Error of route specific

# Structured Covariance MNP (4)

## Variance-Covariance structure in Error term

Error of depend on route length  
 $\epsilon_r = \epsilon_r^1 + \epsilon_r^0$   
Error of route specific

$$\Sigma = \Sigma^1 + \Sigma^0$$

$$\Sigma = \sigma^2 \begin{pmatrix} L_1 & L_{12} & \cdots & L_{1R} \\ L_{12} & L_2 & \cdots & L_{2R} \\ \vdots & \vdots & \ddots & \vdots \\ L_{1R} & L_{2R} & \cdots & L_R \end{pmatrix} + \sigma_0^2 I$$

Simplify use cov. ratio

$$\Sigma = \sigma_0^2 \begin{pmatrix} \eta L_1 + 1 & \eta L_{12} & \cdots & \eta L_{1R} \\ \eta L_{12} & \eta L_2 + 1 & \cdots & \eta L_{2R} \\ \vdots & \vdots & \ddots & \vdots \\ \eta L_{1R} & \eta L_{2R} & \cdots & \eta L_R + 1 \end{pmatrix}$$

$$\eta = \frac{\sigma^2}{\sigma_0^2}$$

### Error of depend on route length

Variance of route utility increases in proportion to the route length.

$$Var(\epsilon_r^1) = L_r \sigma^2$$

Covariance between routes increases in proportion to the length of route overlap.

$$Cov(\epsilon_r^1, \epsilon_q^1) = L_{rq} \sigma^2$$

### Error of route specific

- independent of each route (cov=0)

$$Cov(\epsilon_r^0, \epsilon_q^0) = \sigma_0^2, \quad q = r$$

$$= 0, \quad q \neq r$$

Estimate only cov. ratio !

$L_r$  : length of route  $r$

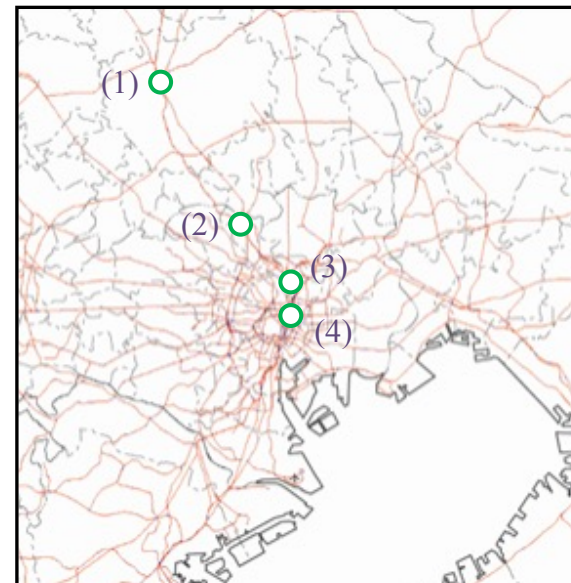
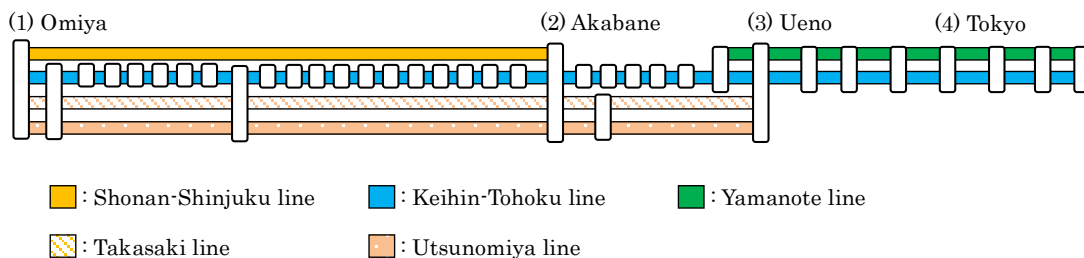
$L_{rq}$  : overlap length between route  $r$  and  $q$

$\sigma^2$  : variance of unit length

# Structured Covariance MNP (5)

Apply to the SCMNL for The 18th master plan for urban railway network in TMA (2000)

Ex : Oomiya to Kanda station



## Estimation results

	parameter	t-value
in-vehicle time	-0.0943	-8.09
access/egress time	-0.127	-11.7
transfer time	-0.112	-10.7
cost	-0.002	-3.98
congestion index	-0.00869	-3.34
$\eta$	0.436	2.71
Adj- $\rho^2$	0.39	
# of sample	1218	

## Prediction results

	Obs	MNL	SCMNP
Utsunomiya + Yamanote	33%	28%	27%
Utsunomiya + Keihin-Tohoku	15%	24%	20%
Keihin-Tohoku	53%	47%	52%

To achieve a high prediction accuracy by the relaxation of route overlap (Obs  $\pm 10\%$  in all route)



## Mixed Logit (Train 2000)

High flexible structure using **two error** term.

Utility function

$$U_i = V_i + \eta_i + v_i$$

$v$  dist.: assume any G function

- IID Gamble (Logit Kernel)  $\Rightarrow$  MNL
- any G function (GEV Kernel)  $\Rightarrow$  NL, PCL, CNL, GNL...

$\eta$  dist.: basically assume “*Normal dist.*”

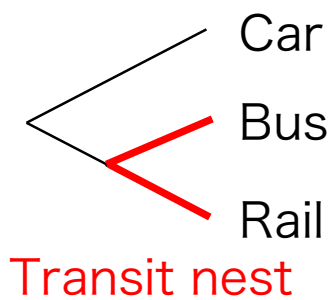
In the case of normal distribution takes a non-realistic value, it can assume a variety of probability distribution (triangular distribution, cutting normal distribution, lognormal distribution, Rayleigh distribution, etc.).

- Error Component: approximate to any GEV model
- Random Coefficient: Consider the heterogeneity

## Approximation of Nested Logit (NL)

Describe the nest (covariance) using structured  $\eta$ .

Ex: model choice



$$\begin{aligned}
 U_{car} &= \beta \mathbf{X}_{car} + v_{car} \\
 U_{bus} &= \beta \mathbf{X}_{bus} + \sigma_{transit} \eta_{transit} + v_{bus} \\
 U_{rail} &= \beta \mathbf{X}_{rail} + \sigma_{transit} \eta_{transit} + v_{rail}
 \end{aligned}$$

Normal  $\Rightarrow$  nest

IID Gamble  $\Rightarrow$  Logit

Choice prob.  
(open-form)

$$P_{rail} = \int_{\eta_{transit}} \frac{e^{V_{rail} + \sigma_{transit} \eta_{transit}}}{e^{V_{car}} + e^{V_{bus} + \sigma_{transit} \eta_{transit}} + e^{V_{rail} + \sigma_{transit} \eta_{transit}}} f(\eta_{transit}) d\eta_{transit}$$



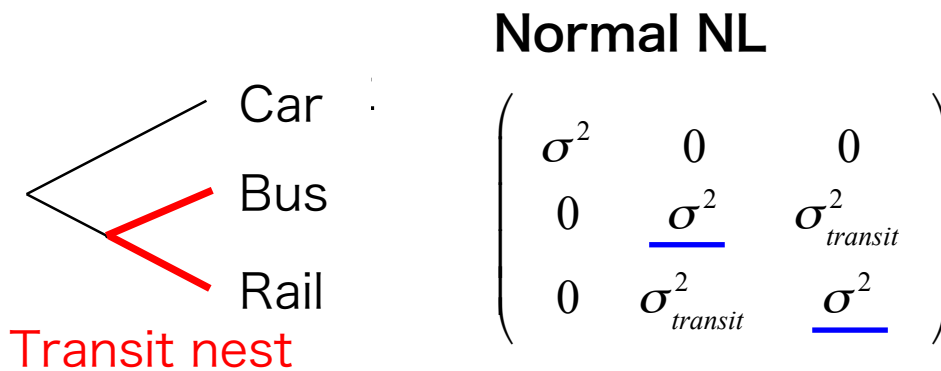
Choice prob.  
(Simulated)

$$P_{rail} = \frac{1}{N} \sum_N \frac{e^{V_{rail} + \sigma_{transit} \eta_{transit}^N}}{e^{V_{car}} + e^{V_{bus} + \sigma_{transit} \eta_{transit}^N} + e^{V_{rail} + \sigma_{transit} \eta_{transit}^N}}$$

$$\eta_{transit} \approx N(0,1)$$

## Approximation of Nested Logit (NL)

Note that variance-covariance matrix is inconsistent with normal NL



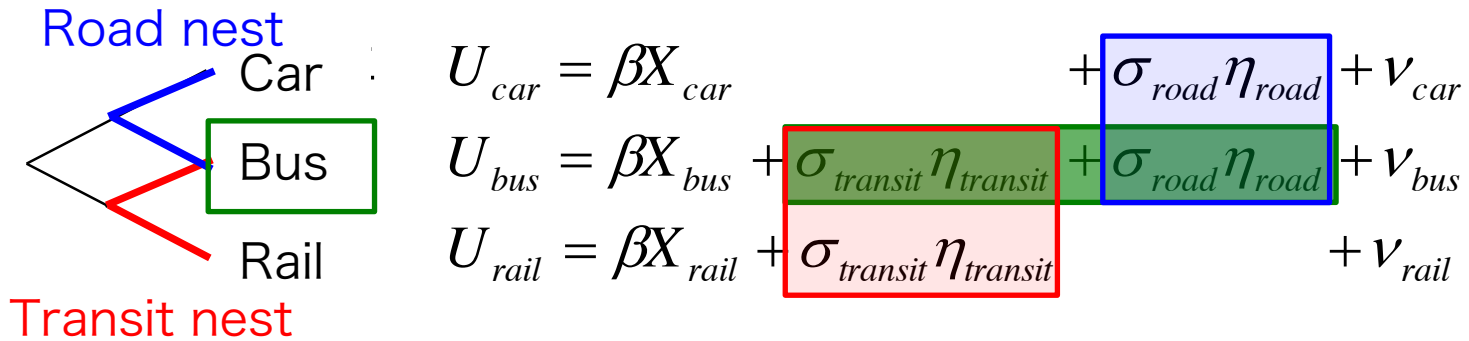
Diagonal elements (variance of Bus and Rail) is bigger than  $\sigma_{transit}$

### Approximated NL based on MXL

$$\begin{matrix} \eta \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{transit}^2 & \sigma_{transit}^2 \\ 0 & \sigma_{transit}^2 & \sigma_{transit}^2 \end{pmatrix} \end{matrix} + \begin{matrix} \nu \\ \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{pmatrix} \end{matrix} = \begin{matrix} \varepsilon \\ \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \underline{\sigma_{transit}^2 + \sigma^2} & \sigma_{transit}^2 \\ 0 & \sigma_{transit}^2 & \underline{\sigma_{transit}^2 + \sigma^2} \end{pmatrix} \end{matrix}$$

## Approximation of Cross Nested Logit (CNL)

Describe the nest (covariance) using structured  $\eta$ .

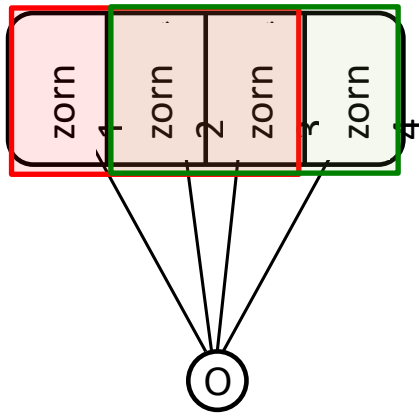


$$\begin{pmatrix} \sigma_{road}^2 & \sigma_{road}^2 & 0 \\ \sigma_{road}^2 & \sigma_{road}^2 + \sigma_{transit}^2 & \sigma_{transit}^2 \\ 0 & \sigma_{transit}^2 & \sigma_{transit}^2 \end{pmatrix} + \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{pmatrix} \quad \eta_{transit}, \eta_{road} \approx N(0,1)$$

$$= \begin{pmatrix} \sigma_{road}^2 + \sigma^2 & \sigma_{road}^2 & 0 \\ \sigma_{road}^2 & \sigma_{road}^2 + \sigma_{transit}^2 + \sigma^2 & \sigma_{transit}^2 \\ 0 & \sigma_{transit}^2 & \sigma_{transit}^2 + \sigma^2 \end{pmatrix} \quad \epsilon$$

## Approximation of Spatial Correlation Logit

Describe the spatial correlation using structured  $\eta$ .



$$\begin{aligned}
 U_{zorn1} &= \beta X_{zorn1} + \sigma \eta_1 + \sigma \eta_2 && + v_{zorn1} \\
 U_{zorn2} &= \beta X_{zorn2} + \sigma \eta_1 + \sigma \eta_2 + \sigma \eta_3 && + v_{zorn2} \\
 U_{zorn3} &= \beta X_{zorn3} && + \sigma \eta_2 + \sigma \eta_3 + \sigma \eta_4 + v_{zorn3} \\
 U_{zorn4} &= \beta X_{zorn4} && + \sigma \eta_3 + \sigma \eta_4 + v_{zorn4}
 \end{aligned}$$

$$\begin{pmatrix}
 \boxed{2\sigma_0^2} & \boxed{\sigma_0^2} & 0 & 0 \\
 \boxed{\sigma_0^2} & \boxed{3\sigma_0^2} & \boxed{\sigma_0^2} & 0 \\
 0 & \boxed{\sigma_0^2} & \boxed{3\sigma_0^2} & \boxed{\sigma_0^2} \\
 0 & \boxed{0} & \boxed{\sigma_0^2} & \boxed{2\sigma_0^2}
 \end{pmatrix}
 +
 \begin{pmatrix}
 \sigma^2 & 0 & 0 & 0 \\
 0 & \sigma^2 & 0 & 0 \\
 0 & 0 & \sigma^2 & 0 \\
 0 & 0 & 0 & \sigma^2
 \end{pmatrix}
 =
 \begin{pmatrix}
 \boxed{\sigma_0^2 + \sigma^2} & \boxed{\sigma_0^2} & 0 & 0 \\
 \boxed{\sigma_0^2} & \boxed{2\sigma_0^2 + \sigma^2} & \boxed{\sigma_0^2} & 0 \\
 0 & \boxed{\sigma_0^2} & \boxed{2\sigma_0^2 + \sigma^2} & \boxed{\sigma_0^2} \\
 0 & \boxed{0} & \boxed{\sigma_0^2} & \boxed{\sigma_0^2 + \sigma^2}
 \end{pmatrix}$$

$$\eta_1, \eta_2, \eta_3 \approx N(0,1)$$

## Approximation of heteroscedastic Logit

Assume the different error variance in each alternatives'

※ **Identification problem** occur in the case of not fixed one of the parameters to zero at least.

$$U_{car} = \beta X_{car} + \sigma_{car} \eta_{car} + v_{car}$$

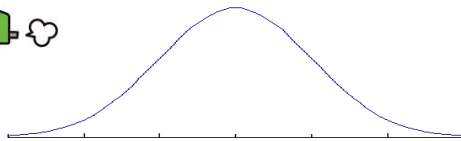
$$U_{bus} = \beta X_{bus} + \sigma_{bus} \eta_{bus} + v_{bus}$$

$$U_{rail} = \beta X_{rail} + \sigma_{rail} \eta_{rail} + v_{rail}$$

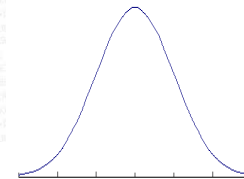
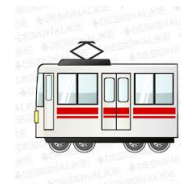
$$\eta_{car}, \eta_{bus}, \eta_{rail} \approx N(0,1)$$

$$\begin{pmatrix} \sigma_{car}^2 + \sigma^2 & 0 & 0 \\ 0 & \sigma_{bus}^2 + \sigma^2 & 0 \\ 0 & 0 & \sigma_{rail}^2 + \sigma^2 \end{pmatrix} \quad \epsilon$$

Assume heteroscedastic in error



**FIX !**  $\sigma_{bus}^2 = 1$



- Car: **Low** travel time reliability  
⇒ Error variance is **large**

- Rail: **High** travel time reliability  
⇒ Error variance is **small**

※ consider only heteroscedastic (IID assumption is not relaxed)

# Random Coefficient (1)

## Taste heterogeneity of decision maker

Parameters defined homogeneously in population. However, decision maker  $n$  has different taste (= heterogeneity)

$$U_{car,n} = \beta T_{car,n} + \varepsilon_{car,n} \quad \rightarrow \quad U_{car,n} = \beta_n T_{car,n} + \varepsilon_{car,n}$$

## Segmentation (observable heterogeneity)

- Constant by gender : male's constant:  $\alpha_0 + \alpha_1$

$$U_{car,n} = \left[ (\alpha_0) + \alpha_1 * male_n \right] + \beta_1 T_{car,n} + \varepsilon_{car,n}$$

Female's constant;  $\alpha_0$

- parameter by gender : male's parameter:  $\beta_1$

$$U_{car,n} = \alpha_0 + \left[ \beta_1 \right] * male_n * T_{car,n} + \left[ \beta_2 \right] * (1 - male_n) * T_{car,n}$$

Female's parameter:

$$\beta_2 * 1 - male_n = female_n$$

## Parameter distribution (unobservable heterogeneity)

Assume the heterogeneity of parameter

⇒ In the case of parameter following Normal dist., we estimate the dist.'s hyper-parameter (mean and variance).

$$U_{car,n} = \beta_n T_{car,n} + v_{car,n}$$

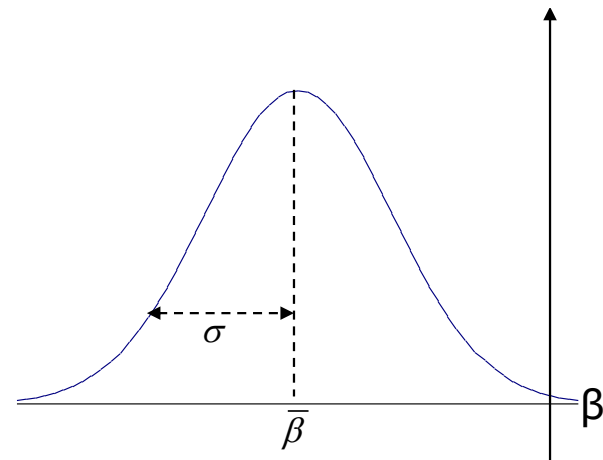
$$\beta_n \approx N(\bar{\beta}, \sigma^2)$$

$$U_{car,n} = \bar{\beta} T_{car,n} + \sigma \eta_n T_{car,n}$$

$$U_{bus,n} = \bar{\beta} T_{bus,n} + \sigma \eta_n T_{bus,n}$$

$$U_{rail,n} = \bar{\beta} T_{rail,n} + \sigma \eta_n T_{rail,n}$$

$$\eta_n \approx N(0,1) \quad \bar{\beta}, \sigma : \text{unknown parameter}$$



Hyper-parameter can describe using observable variables

$$\bar{\beta}_n = \gamma_0 + \gamma_1 \text{income}_n \quad \beta \text{ depend on observable income variable}$$



## Strengths

- ❖ Describe correlation between alternatives' by EC
  - MNP: all alternatives' (relax and reduce by structuring)
  - MXL: depend on approximated model
- ❖ Describe heterogeneity by RC
  - Segmentation, parameter distribution...

## Limitations

- ❖ High calculation cost in parameter estimation
  - Open-form model has high dimensional integration.
  - Recently, proposed high speed estimation methods  
Ex: Bayesian estimation (MCMC) ⇒ see Train's book  
MACML: analytical integration by Bhat et al.(2011)

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# **Discrete choice model**

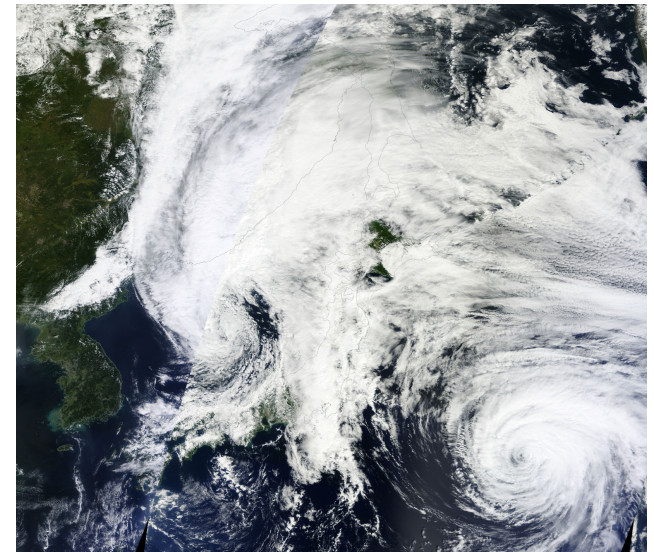
**(Not Supervised Learning, Clustering...)**

## **using sensing data**

# Heavy Rainfall Disaster in Japan

20

## ❖ Tohoku-Kanto Heavy Rainfall (Sep. 9-11, 2015)



### Kinigawa river dike collapsed

Date: Sep. 9-11, 2015

Maximum rainfall: 551mm/day

# of deaths: 14

# of flooded house : over 100,000

Heavy rainfall by huge extratropical cyclone

# Heavy Rainfall Disaster in Japan

21

2015/9/10 11:10



Dike break

Over flow

Flooded Residential Area

## Understanding behavior at disaster time

- Previous study (Behavior at disaster time) is quit limited.
- New transpiration data collection system (called ETC 2.0 probe) is available since 2014
  1. Verify the effectiveness of new data for route choice analysis at disaster time
  2. Apply to the drivers route choice simulation using new data (ETC 2.0) at disaster time

## Analysis target

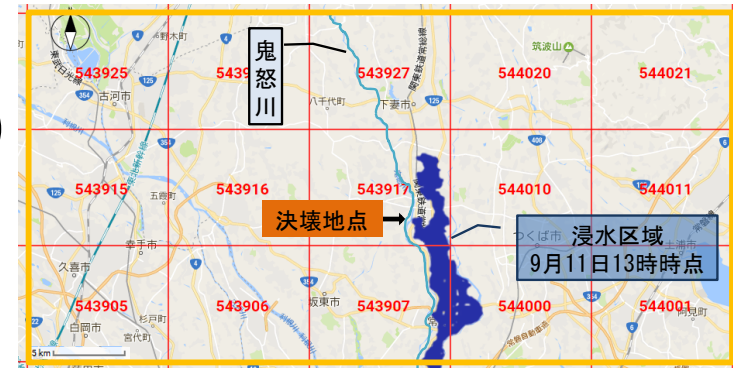
Tohoku-Kanto heavy rainfall (Sep, 2015)

### 【Data set】

Sep. 2015 (1month)

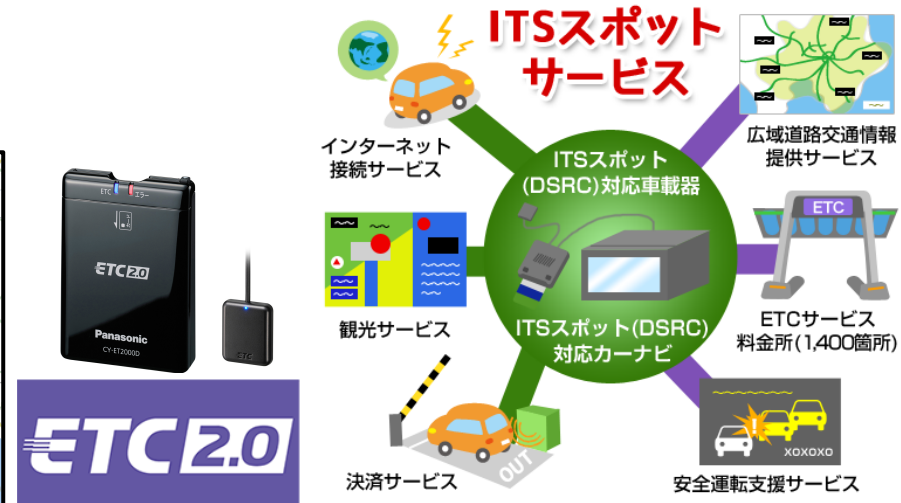
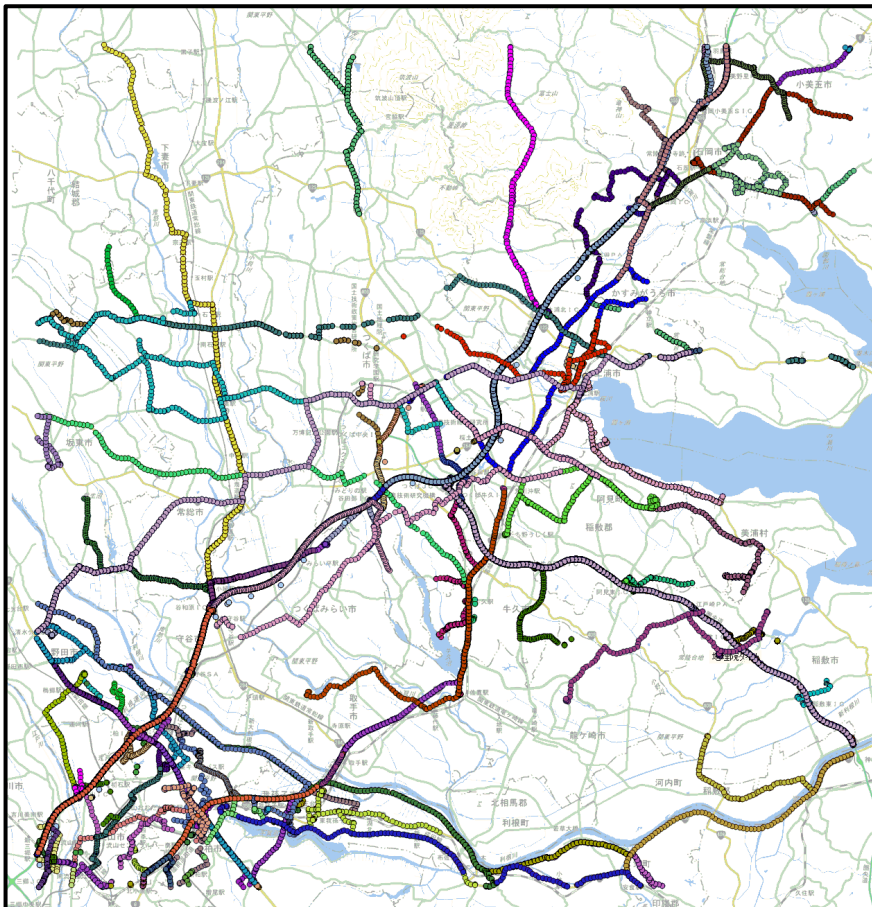
### 【Time series】

Before disaster (normal)	: Sep 1-9, 2015
Rainfall disaster time	: Sep 10-13, 2015
After disaster (normal)	: Sep 14-30, 2015

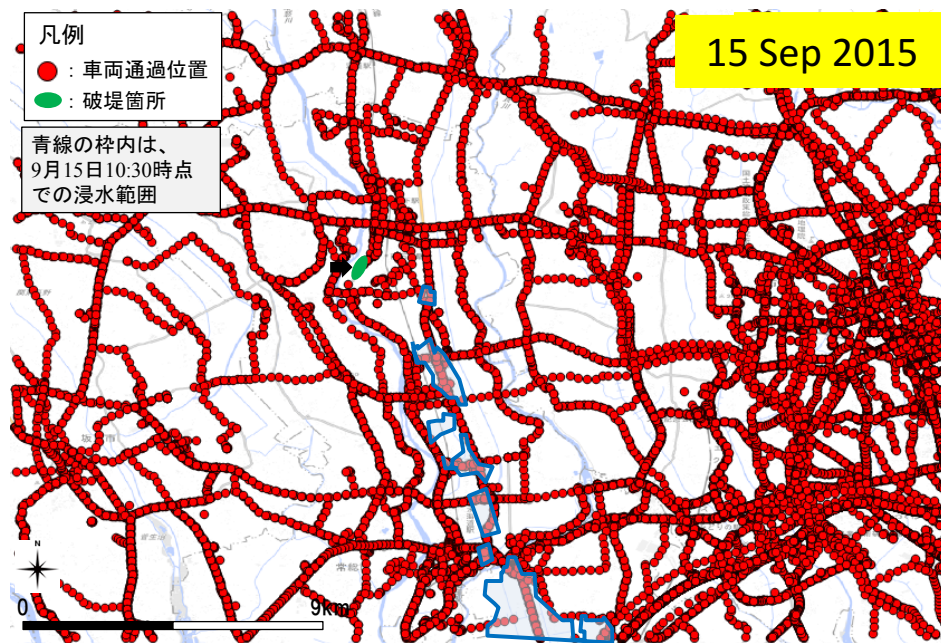
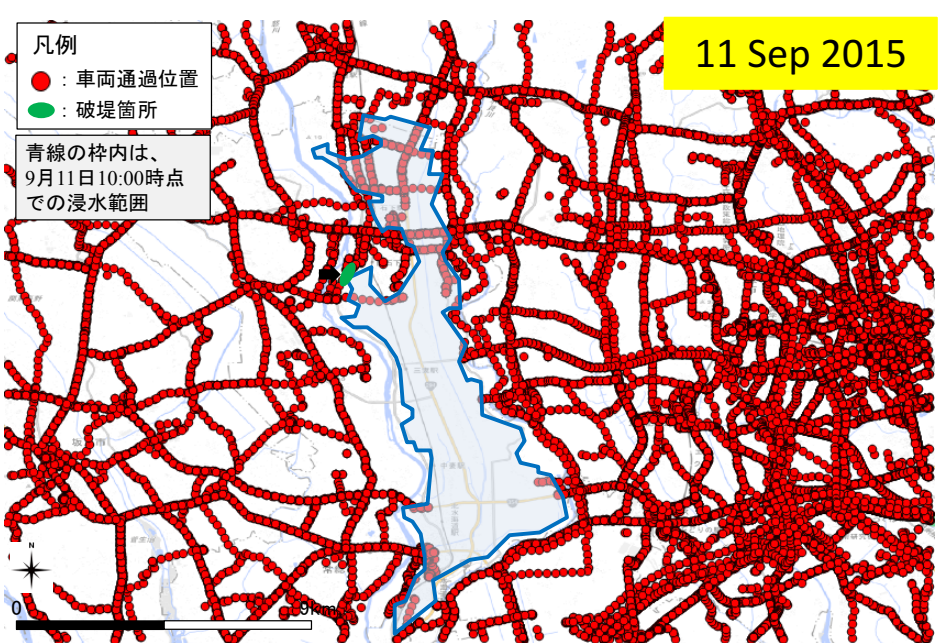
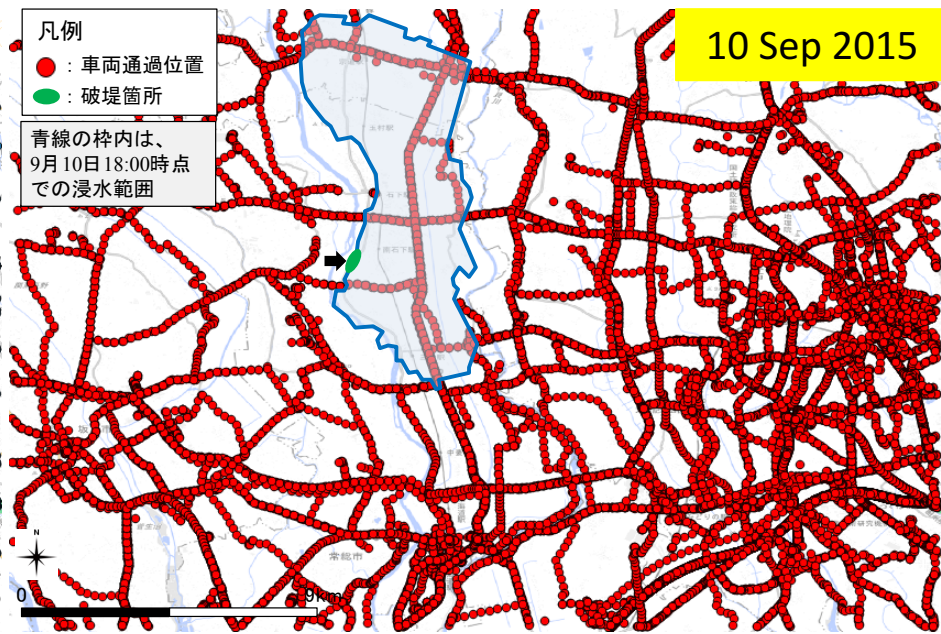
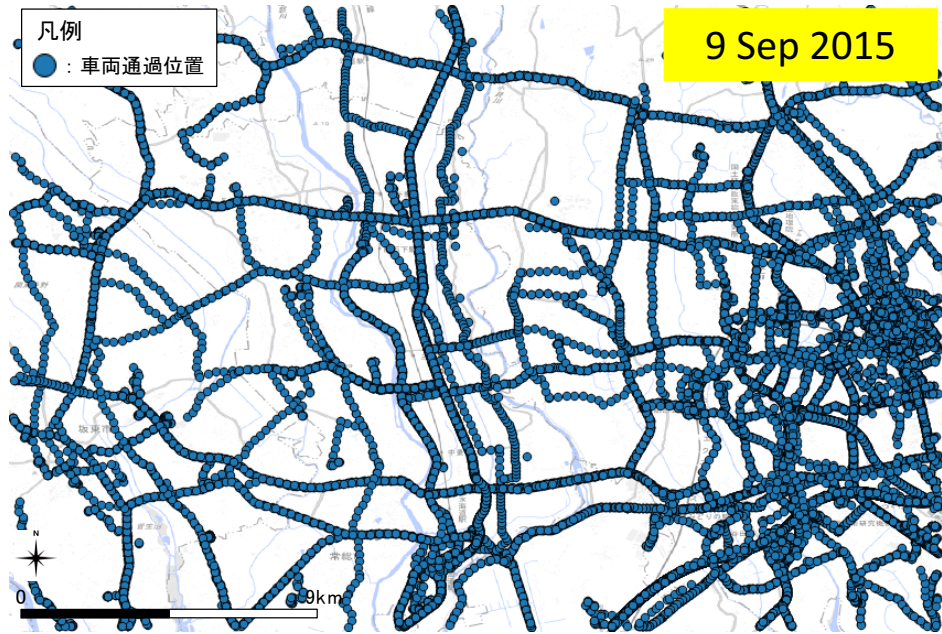


## What is ETC 2.0 ?

- Smart device “Electronic Toll Collection (ETC) + Service”
- This project organized by MLIT
- Device collected vehicle GPS log



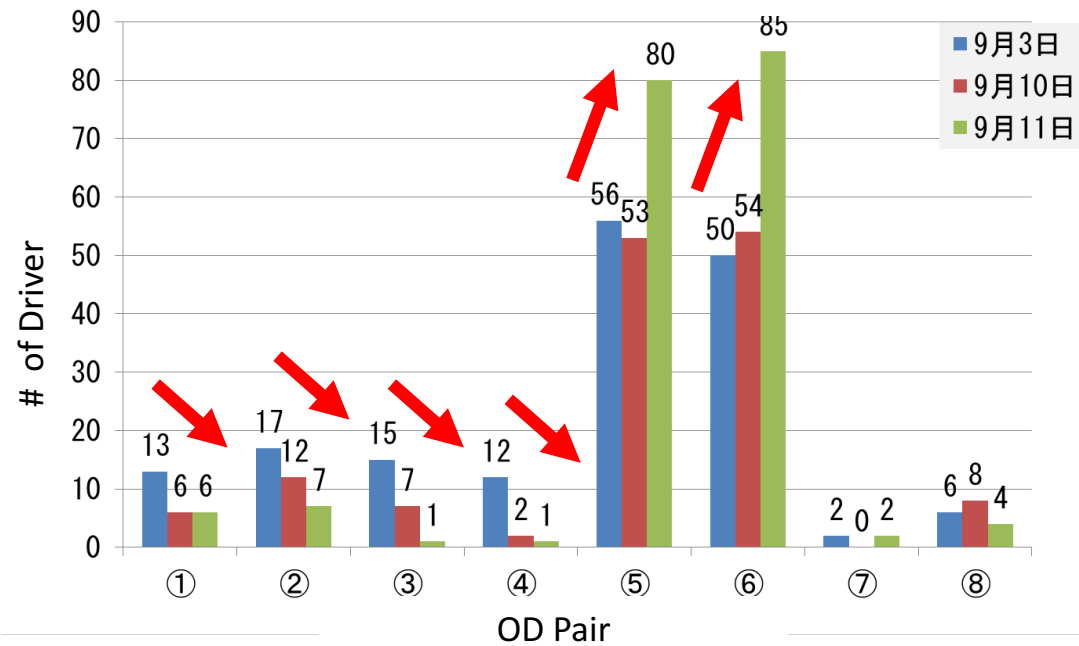
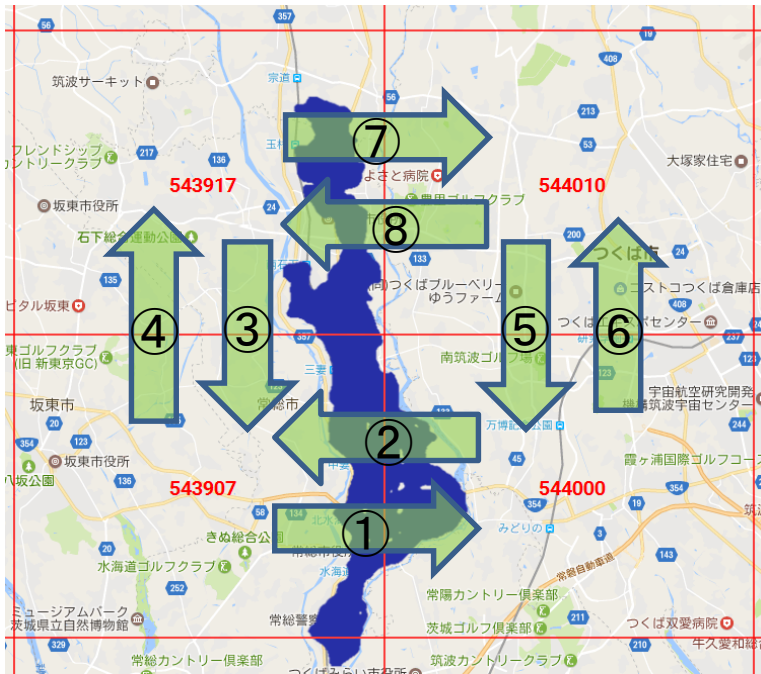
# Probe data in Kinugawa area





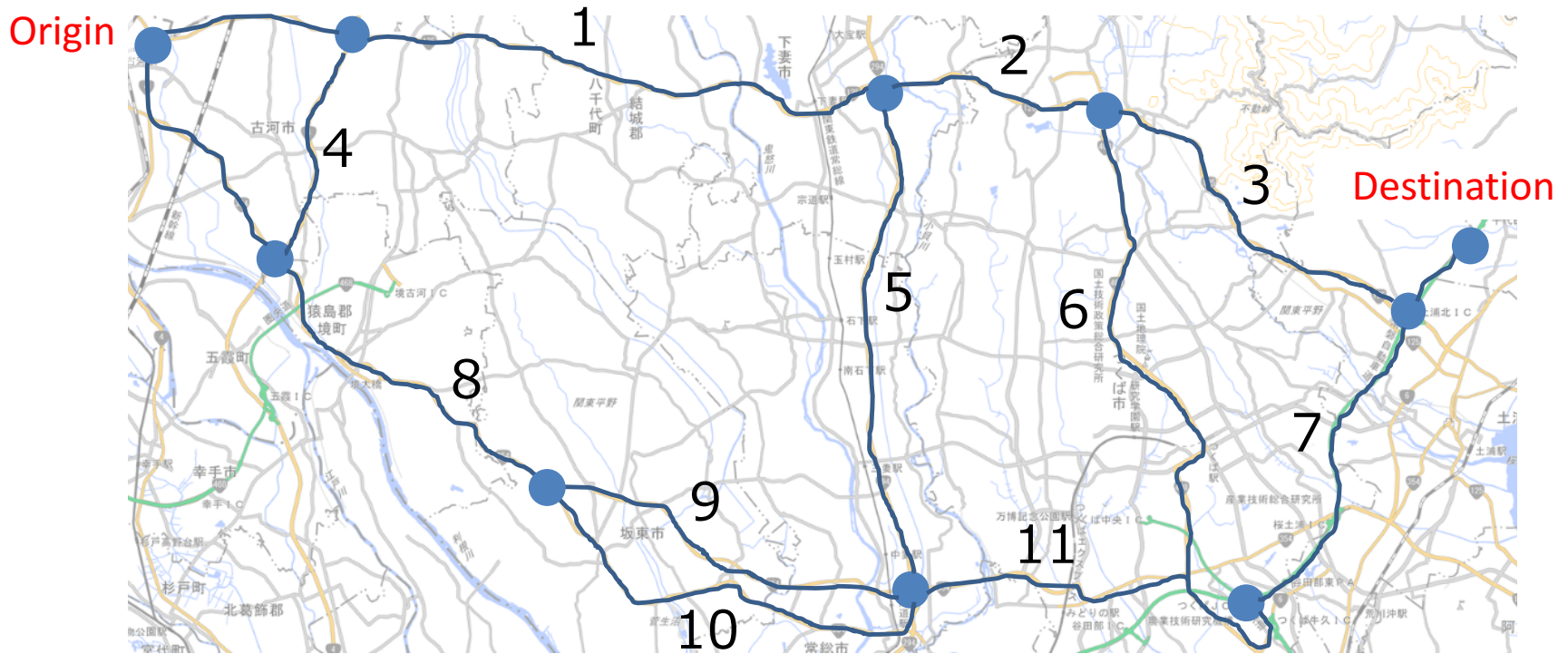
# Route choice: macroscopic

- Aggregate the 8 pattern OD traffic volume in flooded area.
  - Decreasing OD: ①~④ by shut down R294 and R24
  - Increasing OD: ⑤, ⑥ (it is not shut down)
- ⇒ Driver make a detour to avoid a flooded area (1.6 times)



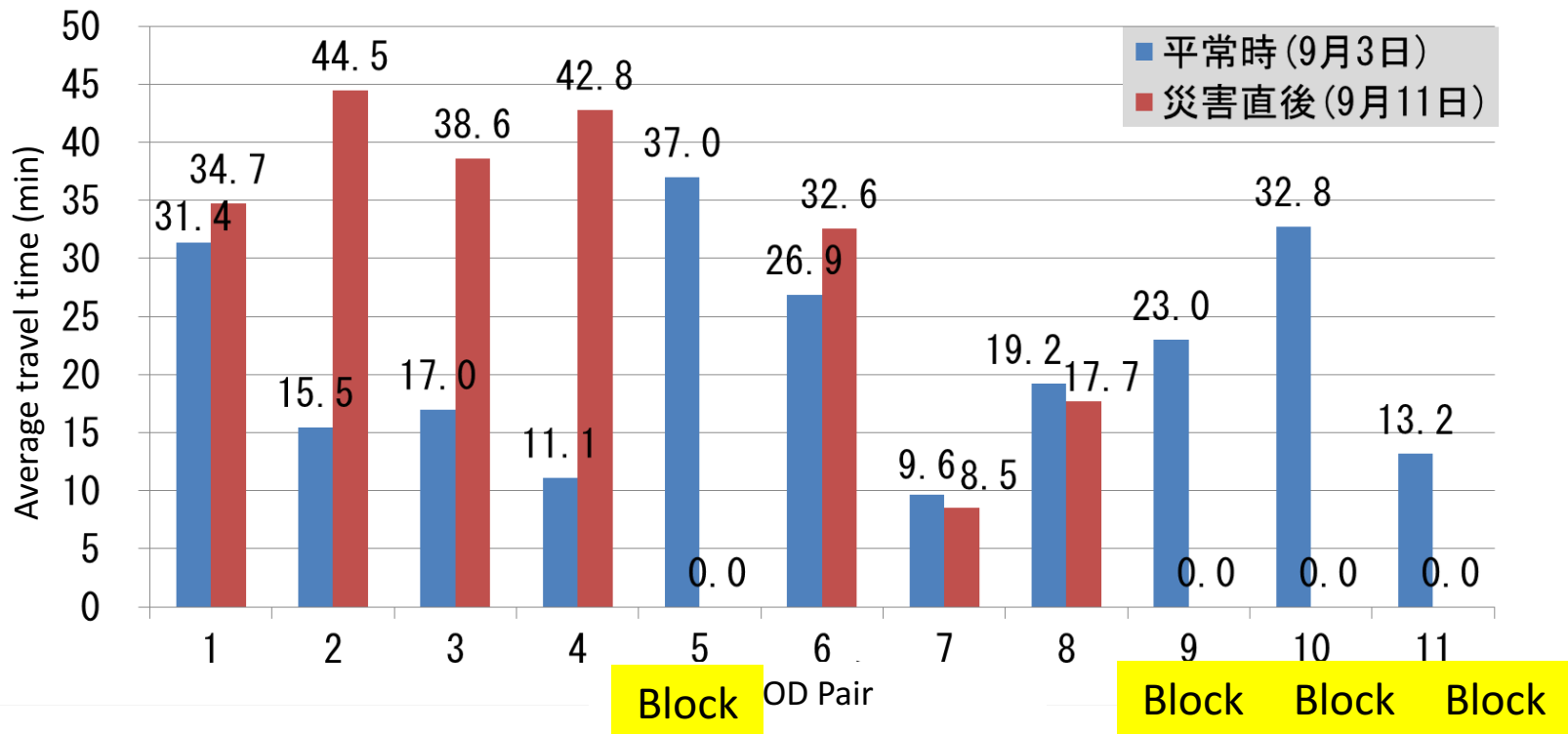
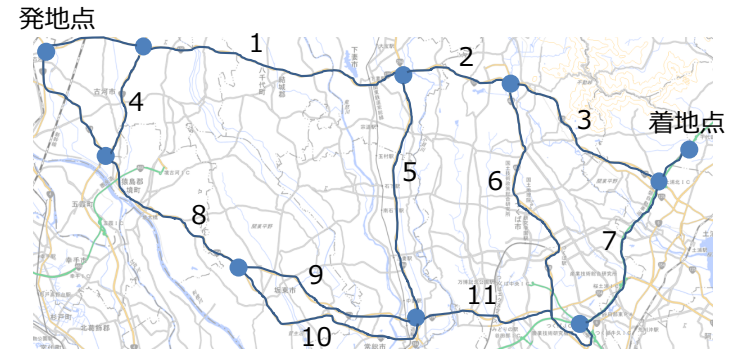
## Road network

- Setting the simple main road network (use only high standard road) in flooded area.



## Travel time

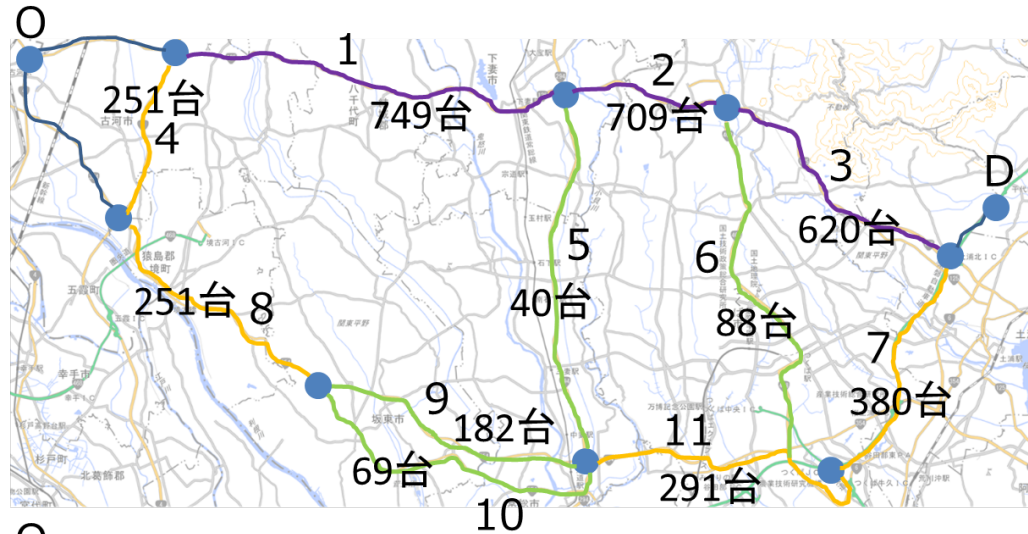
- Setting the average travel time in normal time and rainfall time by ETC 2.0 probe data
- Travel time has Increased 4times



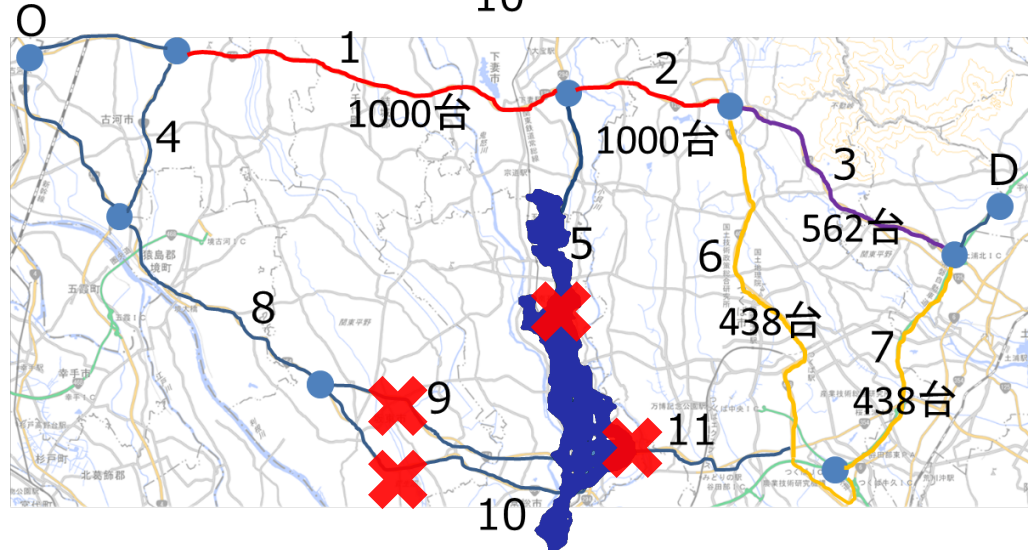
# Route choice simulation

Test route choice simulation assuming OD traffic volume to 1000 units on 3 Sep. 2015 (Normal) and 11 Sep. 2015 (disaster)

## 3 Sep. (Normal)



## 11, Sep. (Rainfall)



## ◆ Conclusions

- We clarified the usability of ETC 2.0 probe data to estimate road network state at disaster time.
- We suggested the possibility of route choice simulation using ETC 2.0

## ◆ Future work (on going)

- Integration of some simulation system  
⇒ Travel activity simulation + urban meteorology simulation
- Real time traffic volume estimation using ETC 2.0
- Apply to large-scale analysis (whole Tokyo area) and multiple disaster simulation (e.g. earthquake & heavy rain)