2015年度 理論談話会 #1 @409

A random utility maximization (RUM) based dynamic activity scheduling model:

Application in weekend activity scheduling

Khandker M. Nurul Habib (2011)

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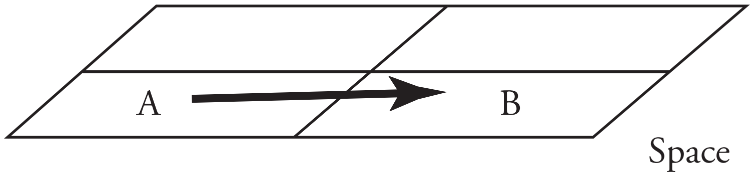
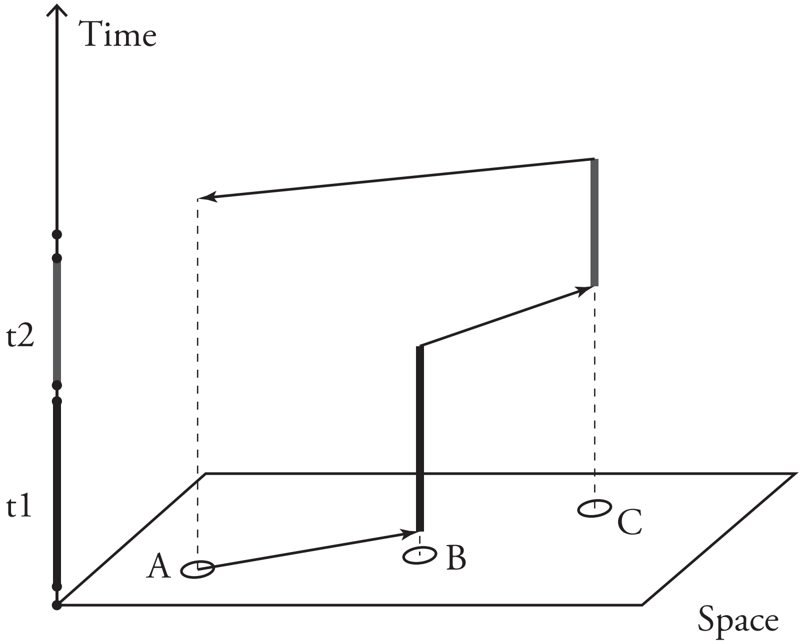
Activity-based approach

- あらゆるスケールの都市政策を評価する上で，発生する**移動・活動需要（from where to where, when）**を把握することは重要.
- 時空間解像度の高い非集計モデルを用いることで，四段階推定法 (1960s) に取って代わるモデルとして期待されている.

・行動：トリップ → 時空間パス
・空間：ゾーン → 任意(施設・セル)
・モデル：集計 → 非集計(個人)





Constrains-based model in *time geography*

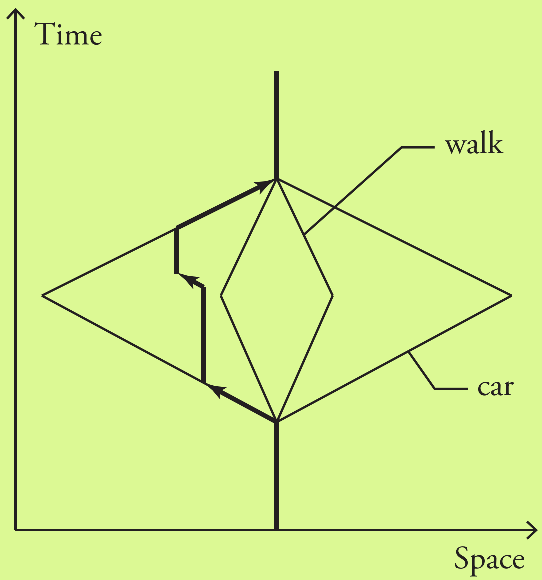
Hagerstrand (1970)

- 個人は空間座標のみならず，時間軸上の位置を持っている.
- 1日（あるいは1週間，1年，1生涯）の行動を，連続した時空間パスとして表現.
- パスは**1）能力の制約，2）結合の制約，3）権威の制約**のもとで描かれる.

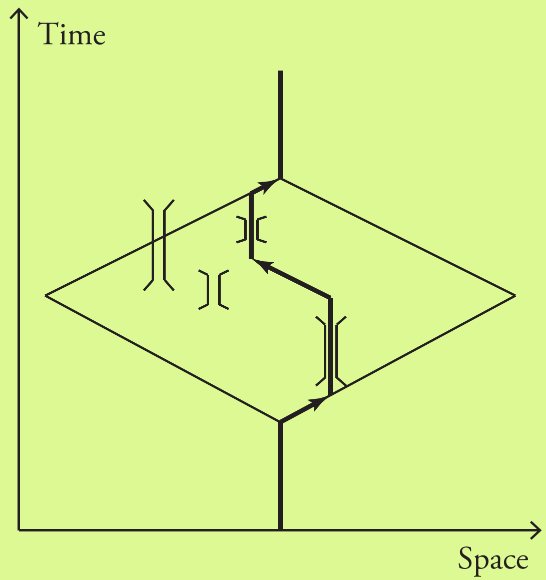
Constrains-based model in *time geography*

Hagerstrand (1970)

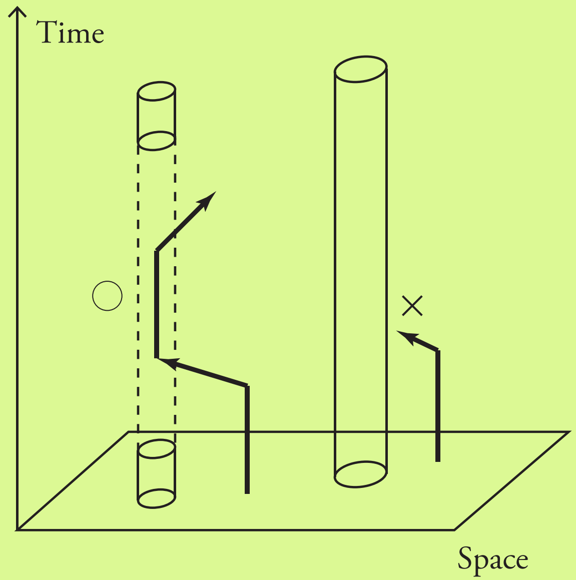
1. 能力の制約：
プリズム (prism)



2. 結合の制約：
バンドル (bundle)



3. 権威の制約：
ドメイン (domain)



- Activity agenda を与え，諸制約を満たすパスとして活動パタンが決定

Pattern choice or Scheduling process ?

Activity agenda

義務活動
・仕事@◯◯
・銀行に行く@△△

非義務活動
・買い物

→

Constraints

prism
・電車+徒歩
・20:00までに帰宅
bundle
・9:00-17:00は会社
domain
・銀行は9:00-15:00

→

Which is the best ?

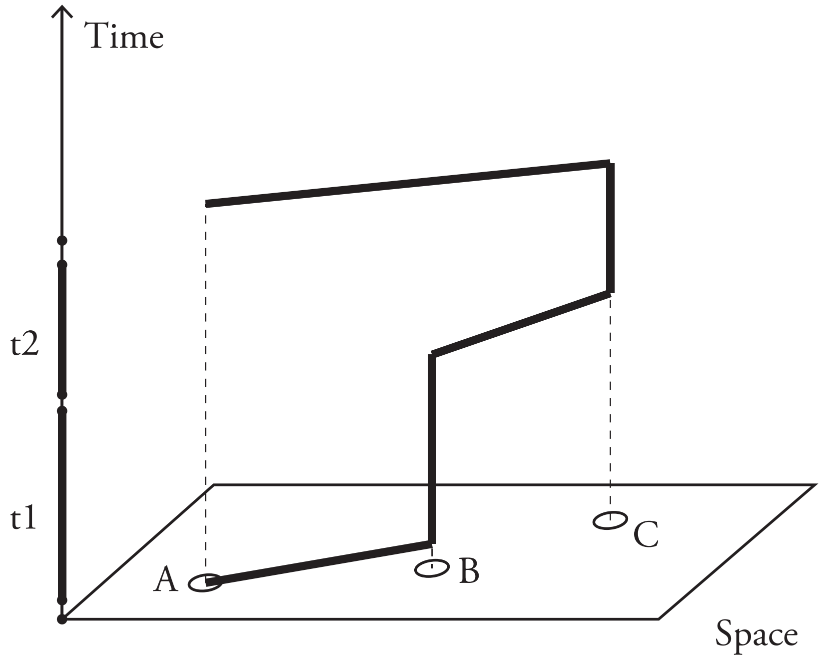
Pattern 1
9:00 会社→ 12:00 銀行 →17:00 会社→ 17:30 買い物→ 19:00 帰宅
Pattern 2
会社→・・・
Pattern 3
会社→・・・

- Pre-trip 型の意思決定モデル
- 余暇や回遊などのより自由な活動パタンには…？

Pattern choice or Scheduling process ?

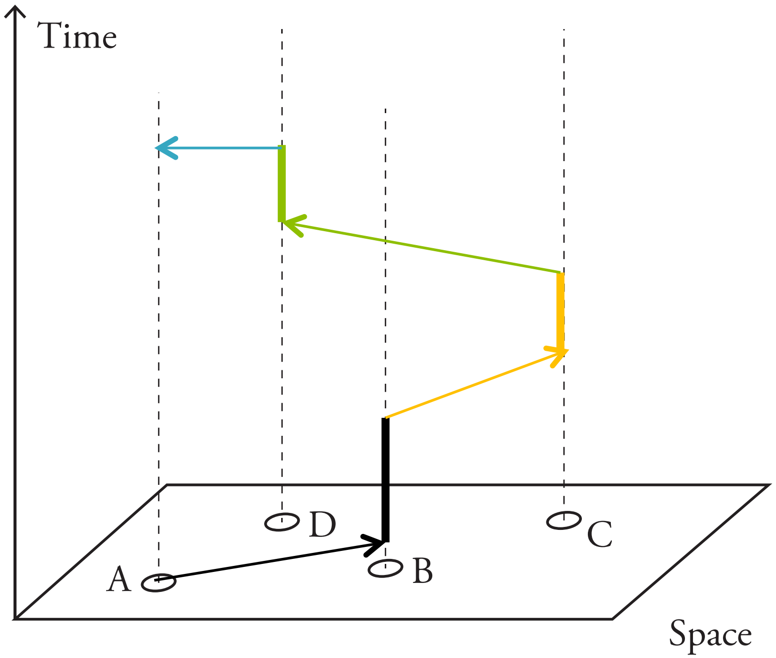
Utility-maximizing model
(Pre-trip decision)

Bowman and Ben-Akiva (2001),
Recker (1995)



Scheduling process model
(Sequential en-route decision)

*Habib (2011)

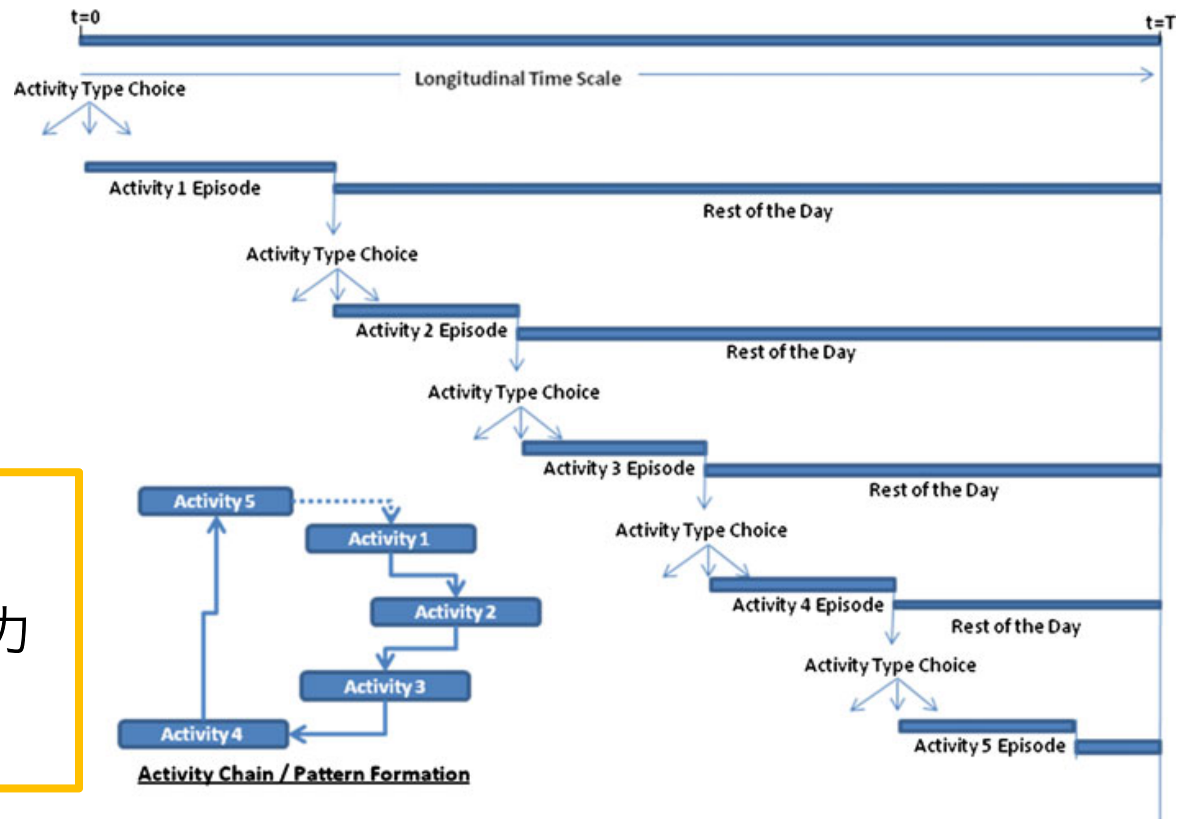


Unplanned activity & Re-scheduling

Dynamic discrete-continuous approach

- 時間制約下で**逐次的に活動種類-活動時間を選択**する離散-連続モデル
- ランダム効用最大化理論（RUM）に基づく

活動パタンは
プロセスの結果として出力
（≠ 選択肢）



Discrete choice model

個人が活動種類 j を選択したときに得られる効用 U_j

$$U_j = V_j + \varepsilon_j = \beta_j x_j + \varepsilon_j; \;\; j = 1,2,\ldots,A \qquad (1)$$

V_j ：効用の確定項

x_j ：説明変数ベクトル

ε_j ：効用の誤差項
(i.i.d. gumbel distribution)

β_j ：重みパラメータベクトル

（ランダム）効用の最も大きい活動種類を選択

活動 j の選択確率

$$\mathrm{P}_j = \Pr\left(U_j > \max_{n \neq j} U_n\right) = \frac{\exp(V_j)}{\exp(V_j) + \sum_{n \neq j} \exp(V_n)} \qquad (2)$$

Continuous (time expenditure) model

今から行う活動 j と，それ以降行う可能性のある活動集合 c を考える.

活動 j に t_j の時間を使い， t_c 残しておくときの効用 $U(t_k)$

$$U(t_k) = \sum_{k=1}^{2} \frac{1}{\alpha_k} \exp\left(\psi_k z_k + \varepsilon'_k\right)\left(t_k^{\alpha_k} - 1\right) \tag{3}$$

※k=1が活動j，k=2が活動cを表す

α_k ：飽和パラメータ(< 1)*

z_k ：説明変数ベクトル

ε'_k ：効用の誤差項 (i.i.d. gumbel distribution)

ψ_k ：重みパラメータベクトル

RUMに基づく最適行動

時間制約

$$\text{maximize } U(t_k) \quad \text{s.t.,} \quad \sum_{k=1}^{2} t_k = T \tag{4}$$

Continuous (time expenditure) model

最適化問題：

$$\max U(t_k) = \sum_{k=1}^{2} \frac{1}{\alpha_k} \exp\left(\psi_k z_k + \varepsilon'_k\right)\left(t_k^{\alpha_k} - 1\right) \qquad (3)$$

$$\text{s.t.,} \quad \sum_{k=1}^{2} t_k = T \qquad (4)$$

Lagrangian function:

$$l = \underbrace{\sum_{k=1}^{2} \frac{1}{\alpha_k} \exp\left(\psi_k z_k + \varepsilon'_k\right)\left(t_k^{\alpha_k} - 1\right)}_{U(t_k)} - \lambda \underbrace{\left(\sum_{k=1}^{2} t_k - T\right)}_{\text{等式制約}} \qquad (5)$$

KKT条件:

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda = 0 \qquad \text{if } t_k > 0,\ k = 1,2 \qquad (6)$$

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda < 0 \qquad \text{if } t_k = 0,\ k = 1,2 \qquad (7)$$

Continuous (time expenditure) model

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda = 0 \quad \text{if } t_k > 0,\ k = j, c \tag{6}$$

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda < 0 \quad \text{if } t_k = 0,\ k = j, c \tag{7}$$

t_c は非負と仮定すれば，(6)式 (k=c) より，

$$\lambda = \exp\left(\psi_c z_c + \varepsilon'_c\right) t_c^{\alpha_c - 1} \tag{8}$$

$t_j > 0$ のとき，(8)式を(6)式 (k=j) に代入して，

$$\exp\left(\psi_j z_j + \varepsilon'_j\right) t_j^{\alpha_j - 1} = \exp\left(\psi_c z_c + \varepsilon'_c\right) t_c^{\alpha_c - 1} \tag{9}$$

(9)式の両辺の対数をとる.

$$\left\{\psi_j z_j + (\alpha_j - 1)\ln\left(t_j\right)\right\} + \varepsilon'_j = \left\{\psi_c z_c + (\alpha_c - 1)\ln\left(t_c\right)\right\} + \varepsilon'_c \tag{10}$$

Continuous (time expenditure) model

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda = 0 \quad \text{if } t_k > 0,\ k = j, c \qquad (6)$$

$$\exp\left(\psi_k z_k + \varepsilon'_k\right) t_k^{\alpha_k - 1} - \lambda < 0 \quad \text{if } t_k = 0,\ k = j, c \qquad (7)$$

$$\left\{\psi_j z_j + (\alpha_j - 1)\ln\left(t_j\right)\right\} + \varepsilon'_j = \left\{\psi_c z_c + (\alpha_c - 1)\ln\left(t_c\right)\right\} + \varepsilon'_c \qquad (10)$$

ここで,

$$V'_k = \psi_k z_k + (\alpha_k - 1)\ln\left(t_k\right) \qquad (11)$$

とすれば（ $t_j = 0$ のときも同様にして）次式を得る.

$$V'_j + \varepsilon'_j = V'_c + \varepsilon'_c \quad \text{if } t_j > 0 \qquad (12)$$

$$V'_j + \varepsilon'_j < V'_c + \varepsilon'_c \quad \text{if } t_j = 0 \qquad (13)$$

Continuous (time expenditure) model

$$\varepsilon'_j - \varepsilon'_c = V'_c - V'_j \quad \text{if } t_j > 0 \qquad (12)$$

$$\varepsilon'_j - \varepsilon'_c < V'_c - V'_j \quad \text{if } t_j = 0 \qquad (13)$$

$t = t_j$　となる確率

$$\Pr(t = t_j) = \left(\frac{1-\alpha_j}{t_j} + \frac{1-\alpha_c}{t_c} \right) \cdot \frac{1}{\sigma} \exp\left(\frac{-(V'_c - V'_j)}{\sigma} \right) \cdot \left[1 + \exp\left(\frac{-(V'_c - V'_j)}{\sigma} \right) \right]^{-2} \qquad (14)$$

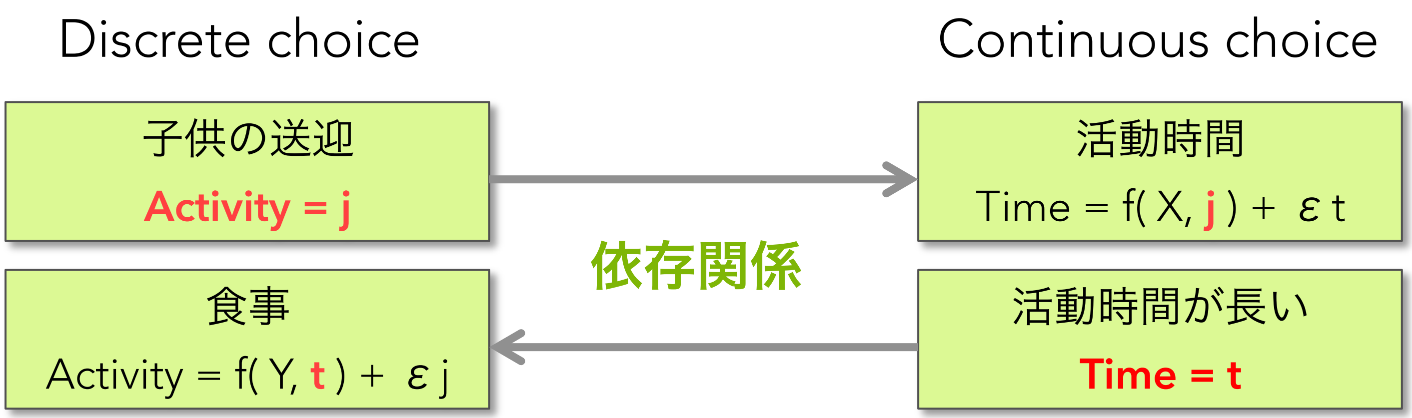
$\dfrac{\delta\left((V'_c - V'_j)\right)}{\delta t_j}$

$\Pr(\varepsilon'_j - \varepsilon'_c < V'_c - V'_j)$

※ガンベル分布に従う2つの確率変数の差はロジスティック分布に従う.

Discrete-continuous model

- RUMに基づく離散・連続選択それぞれの選択確率を導出
- 別々に推定していいのか？→ **Self-selection, 内生性**



- 部分的な共通要因によって関連づけられている状況を記述

離散-連続モデル

Joint probability of discrete-continuous choice

活動種類と活動時間の**同時確率分布**を求めたい.

$$\Pr(Time = t_j \cap Activity = j)$$

$$\sim \mathrm{BVN}[J_1(\varepsilon_j), J_2(\varepsilon'_j), \rho_{jt}]$$

二変量正規分布

準備：誤差分布を標準正規分布に変換 (Lee, 1983)

$$J_1(\varepsilon_j) = \Phi^{-1}[(\varepsilon_n - \varepsilon_j) < (V_j - V_n)]$$
$$J_2(\varepsilon'_j) = \Phi^{-1}[(\varepsilon'_j - \varepsilon'_c) < (V'_c - V'_j)] \quad (15)$$

※CDFの逆関数の意味

input	$\varepsilon^* \sim N(0,1)$	→	Φ	→	$P(\varepsilon < \varepsilon^*)$	output
	$P(\varepsilon < \varepsilon^*)$	→	Φ^{-1}	→	$\varepsilon^* \sim N(0,1)$	

Joint probability of discrete-continuous choice

$$\Pr(Time = t_j \cap Activity = j) = \Pr(t = t_j \cap \varepsilon \le J_1(\varepsilon_j))$$

$$= \underbrace{\left(\frac{1-\alpha_j}{t_j} + \frac{1-\alpha_c}{t_c}\right) \cdot \frac{1}{\sigma} \exp\left(\frac{-(V'_c - V'_j)}{\sigma}\right) \cdot \left[1 + \exp\left(\frac{-(V'_c - V'_j)}{\sigma}\right)\right]^{-2}}_{\Pr(t = t_j)}$$

$$\times \underbrace{\Phi\left(\frac{J_1(\varepsilon_j) - \rho_{jt} J_2(\varepsilon'_j)}{\sqrt{1-\rho_{jt}^2}}\right)}_{\Pr(\varepsilon \le J_1(\varepsilon_j) \mid \varepsilon' = J_2(\varepsilon'_j))} \qquad (16)$$

尤度関数

個人　活動回数　活動種類

$$L = \prod_{i=1}^{N} \left[\prod_{z=1}^{S} \left[\prod_{j=1}^{A} \left(\Pr_i(Time = t_j \cap Activity = j) \right)^{D_{ij}} \right]_z \right] \qquad (17)$$

（ D_{ij} ：個人 i が活動 j を選択していれば 1，それ以外 0）

Joint probability of NL-continuous model

例えば，活動種類-活動場所選択のNested Logit model

$$\Pr(\text{Activity} = j \ \& \ \text{Location} = l)$$

MNLと同様に，誤差分布を標準正規分布に変換

$$J_1(\varepsilon_{jl}) = \Phi^{-1}[(\varepsilon_{nm} - \varepsilon_{jl}) < (V_{jl} - V_{nm})]$$
$$J_2(\varepsilon'_j) = \Phi^{-1}[(\varepsilon'_j - \varepsilon'_c) < (V'_c - V'_j)] \qquad (15)'$$

離散(NL)-連続の同時確率分布は,

$$\Pr(t_j \cap j \,\&\, l) =$$
$$\left(\frac{1-\alpha_j}{t_j} + \frac{1-\alpha_c}{t_c}\right) \cdot \frac{1}{\sigma} \exp\left(\frac{-(V'_c - V'_j)}{\sigma}\right) \cdot \left[1 + \exp\left(\frac{-(V'_c - V'_j)}{\sigma}\right)\right]^{-2} \times \Phi\left(\frac{J_1(\varepsilon_{jl}) - \rho_{jlt} J_2(\varepsilon'_j)}{\sqrt{1-\rho_{jlt}^2}}\right) \qquad (16)'$$

Application | Weekend activity scheduling

Data for estimation

- 2002-2003 in Toronto,　426 individuals in 264 households
- (平均) 世帯構成員数: 3, 子供の数: 0.6, 居住年数: 18, 収入: 47,000 CAD
- 活動種類(MNL)-活動時間選択モデルを推定

Estimation results

その１(離散選択部分)

- 経過時間が活動選択に影響
- 送迎/買い物/サービスなどは早い時間帯に行われる→優先度が高い

※活動は全９分類

Activity Type 1: *Basic Need* type activities, such as sleep, wash/dress/pack/snack, lunch/dinner/breakfast and other basic need type activities.
Activity Type 2: *Work/School* type activities including telework, volunteer work, training etc.
Activity Type 3: *Household Obligations* type activities, such as cleaning, maintenance, attending children, attending pets and other household obligations.
Activity Type 4: *Drop-off/Pick-up* type activities dropping/picking people, meals, snacks, video rentals, dry cleaning, mails, etc.
Activity Type 5: *Shopping* activities including grocery shopping, personal shopping etc.
Activity Type 6: *Service* type activities, such as medical/hospital, personal beautification, banking, banking, religious service, servicing automobile, etc.
Activity Type 7: *Recreational/Entertainment* type activities.
Activity Type 8: *Social* activities including visiting, hosting, bars/clubs, sports, planned social events, long time telephone conversation, etc.
Activity Type 9: *Other* activities that do not fall in the above mentioned categories.

Variables	Activity type	Parameter	t-Statistics
Activity type choice model component			
Constant			
	Household obligations	1.3574	1.485
Star time in hours from mid-night			
	Drop-off/pick-up	−0.3584	−8.663
	Shopping	−0.1556	−2.921
	Services	−0.3429	−5.808
	Recreation/entertainment	0.0123	0.807
Number of activities already performed from beginning of the day			
	Basic needs	−0.0687	−4.894
	Drop off/pick up	0.2110	3.915
	Shopping	0.1746	3.008
	Services	0.2178	2.836
	Social	0.0721	2.466
Total travel time (minutes)			
	Basic needs	−0.1755	−9.496
	Work/school	−0.0221	−1.529
	Household obligations	−0.1478	−8.324
	Recreation/entertainment	−0.1571	−7.886
	Other	−0.0682	−3.095
Household size: number of people in the household			
	Work/school	−0.1775	−1.993
	Recreation/entertainment	−0.1281	−2.531
	Social	−0.2404	−2.406
	Other	−0.2591	−2.516
Logarithm of age in years			
	Work/school	−0.5769	−6.688
	Household obligations	−0.5350	−2.21
	Shopping	−0.4803	−4.333
	Recreation/entertainment	−0.2723	−3.539
	Social	−1.0267	−5.233
	Other	−0.3649	−2.709
Logarithm of yearly income in Canadian dollars (2002–2003)			
	Household obligations	−0.0503	−3.152
	Services	−0.1296	−5.294
	Social	0.0332	0.868
	Other	−0.0646	−1.916
Employment status: non-full time job			
	Shopping	−0.2498	−0.893
	Recreation/Entertainment	0.2199	1.164
	Social	0.9300	2.555
Number of children in household			
	Household obligations	0.0850	1.553

Application | Weekend activity scheduling

Variables	Activity type	Parameter	t-Statistics
	Drop off/pick up	0.0929	1.273
	Shopping	−0.4030	−3.225
Time expenditure model component			
Variance (Sigma)		0.4952	26.354
Constant			
	Basic needs	0.8195	1.273
	Social	1.8545	−3.225
	Other	3.9394	26.354
Star time in hours from mid night			
	Work/school	−0.0248	−0.923
	Household obligations	0.1071	6.749
	Drop off/pick up	0.0729	2.31
	Shopping	0.0682	2.955
	Services	0.0602	2.483
	Recreation/entertainment	0.0375	0.954
Number of activities already performed from beginning of the day			
	Basic needs	0.0471	3.456
	Social	−0.0343	−1.184
Total travel time (minutes)			
	Household obligations	−0.0170	−1.558
	Drop off/pick up	0.0275	1.926
	Shopping	0.0382	2.832
	Services	0.0249	1.321
	Social	0.0295	2.962
	Other	0.0340	2.567
Household size: number of people in the household			
	Work/school	0.0557	0.727
	Household obligations	0.2049	4.758
	Recreation/entertainment	0.0868	2.471
Logarithm of age in years			
	Work/school	0.4589	3.005
	Drop off/pick up	−0.3269	−2.597
	Recreation/entertainment	0.4949	3.882
	Social	−0.2097	−0.753
	Other	−0.6673	−1.329
Logarithm of yearly income in Canadian dollars (2002–2003)			
	Work/school	0.0466	1.674
	Household obligations	0.0207	1.64
Number of automobile in household			
	Household obligations	−0.1242	−1.224
Logarithm of duration (years) of living in the city			
	Household obligations	−0.1306	−2.655
	Recreation/entertainment	−0.1013	−1.875

Variables	Activity type	Parameter	t-Statistics
	Other	0.2185	1.573
Number of children in household			
	Household obligations	−0.1027	−2.138
Satiation parameter			
Constant			
	Household obligations	−0.1383	−2.36
	Drop off/pick up	−0.0427	−0.42
	Shopping	−0.1093	−1.21
	Recreation/entertainment	−0.4619	−5.51
	Social	−0.2111	−1.83
	Other	−0.2455	−2.09
	Composite activity	1.2654	9.99
Continuous start time in hours from mid night			
	Recreation/entertainment	0.0234	3.74
	Social	0.0173	2.45
Start hour: time of the day			
	Composite activity:		
	Before 6 AM	−0.8773	−8.40
	6:01 AM to 7 AM	−0.2232	−2.13
	7:01 AM to 8 AM	−0.1707	−1.82
	8:01 AM to 9 AM	−0.1693	−1.90
	9:01 AM to 10 AM	−0.2023	−2.32
	10:01 AM to 11 AM	−0.2079	−2.45
	11:01 AM to 12 noon	−0.2214	−2.74
	12:01 noon to 1 PM	−0.1891	−2.46
	1:01 PM to 2 PM	−0.1500	−1.99
	2:01 PM to 3 PM	−0.0514	−0.70
	3:01 PM to 4 PM	−0.0714	−1.03
	4:01 PM to 5 PM	−0.0712	−1.06
	After 5 PM	–	–
Correlation coefficient between activity type choice and time expenditure			
Constant		−0.4423	−3.43
Loglikelihood of full model			−8595.7618
Loglikelihood of constant-only model			9877.646
Adjusted Rho-square value			0.12

α_k

- 活動cでαが正→出来る限り残しておきたい（多様な活動をしたい傾向）

Appendix

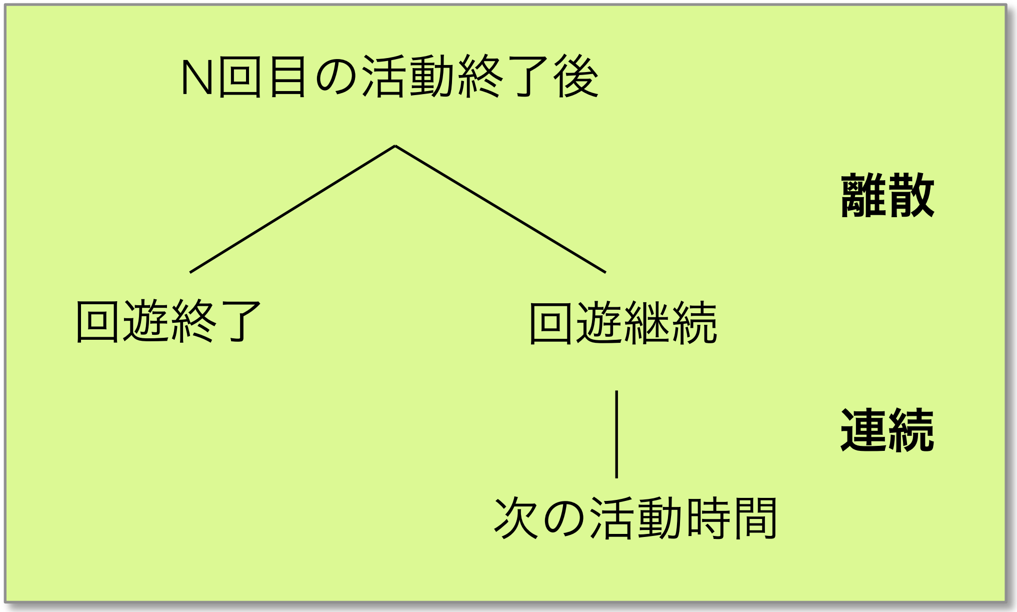
Structural / Reduced-form

*福田・力石(2012)

- **構造型**：ミクロ経済理論との整合性を第一義．ロワの恒等式/KKT条件より演繹的に導出． → 今回のモデル，MDCEV(Bhat, 2005) など
- **誘導型**：現象の直接的記述に主眼を置く統計モデル．不完全観測下のモデルに有効． → Tobit model type I~V など

回遊モデルで比較してみる…

1. Type II tobit model (Heckman, 1979)
2. Habib (2011) 型



Appendix

		Type II Tobit model		Habib(2011) model	
	Variable	Parameter	t-value	Parameter	t-value
	Discrete choice				
α_1	Constant	-0.087	-0.25	-0.382	-0.63
α_2	Female dummy	0.339	1.98 *	0.560	1.88 *
α_3	Elderly dummy	-0.577	-2.03 *	-0.931	-1.95 *
α_4	Log(EP-Main dist.(km) + 1)	0.218	5.48 **	0.369	5.12 **
α_5	Car inflow dummy	-0.213	-1.31	-0.356	-1.30
β_{time1}	Basic parameter of time (min./10)	-0.457	-3.73 **	-0.011	-3.23 **
β_{time2}	A Number of trips	0.104	1.58	0.002	1.14
β_{time3}	Shopping dummy	-0.456	-2.52 **	-0.014	-2.70 **
β_{walk}	Cumulative walking dist. (m/100)	1.626	3.49 **	2.985	3.77 **
β_{walk2}	(Cumulative walking dist.)^2	-0.939	-3.58 **	-1.673	-3.66 **
γ_1	Cumulative trips	-0.497	-4.20 **	-0.773	-3.86 **
γ_2	Cumulative shopping trips	0.395	3.20 **	0.689	3.32 **
γ_4	Previous activity : eating	0.642	3.05 **	0.922	2.58 **
γ_5	Previous activity : Main	-0.827	-4.57 **	-1.285	-4.25 **
γ_6	Shopping street	0.479	2.48 *	0.625	1.87
	Continuous choice				
β_{time1}	Constant	0.958	8.25 **	6.004	2.87 **
β_{time2}	Previous activity time (min./10)	0.389	5.87 **	0.017	4.67 **
β_{time3}	Previous activity: shopping	-0.227	-2.26 *	-0.705	-2.49 *
β_{time4}	Previous activity: Main	-0.521	-4.30 **	-1.304	-3.35 **
β_{time5}	Log(EP-dist. (km))	-0.034	-2.09 *	-0.151	-3.38 **
Log(σ)	Std. deviation of error term	-0.456	-7.06 **	1.000	-
ϱ	Correlation	-0.590	-2.72 **	0.665	2.27 *
αj	Satiation Parameter of activity j	-	-	-0.773	-7.85 **
αc	Satiation Parameter of activity c	-	-	0.880	0.83
	Observations		482		482
	Initial Likelihood		-549.843		-1173.584
	Final Likelihood		-334.971		-935.792
	Rho square (adj.)		0.351		0.183

プリズム(資源制約)があるので時間のパラメータが小さい?

Correlation：正負が逆転（解釈が難しい…）

Satiation：残しておく方が効用の時間減衰率が低い

Summary

- 1日の活動スケジュール決定プロセスを，逐次離散-連続モデルによって構築
- ミクロ経済理論(RUM)をベースに，KKT条件から選択確率を導出(構造型モデル)

自分の研究について

- 活動場所–活動種類選択をNLにしてやれば状態変数の推移確率を内生的に求められる！
- アクティビティパス選択モデルを pre-trip / en-route 型で比較したい (スライド5) .

Reference

※補足の内容は以下の論文を参考にしています.

1. Bhat, C.R.: A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions. Transp. Res. Part B 39(8), 679–707, 2005.
2. Bhat, C.R.: The Multiple Discrete-Continuous Extreme Value (MDCEV) model: role of utility function parameters, identification considerations, and model extensions. Transp. Res. Part B 42, 274–303. 2008.
3. Bowman, J. L., Ben-Akiva, M. E.: Activity-based dis-aggregate travel demand model system with activity schedules, Transportation Research A, Vol. 35, No, 1, pp. 1-28, 2001.
4. Hagerstrand, T.: What about people in regional science? Papers of the Regional Science Association, Vol. 24, No. 1, pp. 6-21, 1970.
5. Heckman, J.J.: Sample selection bias as a specification error, Econometrica 47, 153-161, 1979.
6. Lee, L.-F.: Generalized econometric models of selectivity. Econometrica 51, 507–512, 1983.
7. Recker, W.W.: The household activity pattern problem: general formulation and solution. Transportation Research Part B 29 (1), 61–77, 1995.
8. 福田大輔，力石真：離散-連続モデルの研究動向に関するレビュー，土木学会論文集D3, Vol.69, No.5, pp.497-510, 2013.